MODIFIED RAMSEY DISCOUNTING FOR CLIMATE CHANGE

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Abstract

The Ramsey rule for the consumption rate of discount assumes a transfer of money of a (representative) agent at one point in time to the same agent at another point in time. Climate policy transfers money not just over time but also between agents. We propose three alternative modifications of the Ramsey rule to account for this. Taking a Ramsey rule as given, we derive an intuitively clear but ad hoc modification. Using the assumptions underlying the Ramsey rule, we derive a consistent but more elaborate modification. If the discount rate is differentiated by victim, the consistent modified Ramsey rule is simpler and identical to regional equity weights. We apply the modified Ramsey rules to estimates of the marginal damage costs of carbon dioxide emissions. The results confirm that optimal climate policy has differentiated carbon taxes. Results also show that the standard Ramsey rule drastically underestimates the social cost of carbon.

Key words

Climate change, social cost of carbon, discount rate, Ramsey rule, equity

1. Introduction

Climate change is a long term problem. The discount rate is therefore a crucial parameter in any economic assessment of the options for climate policy (Arrow et al. 1996). The discount rate is typically set by the Ramsey equation (Ramsey 1928), which has three parameters: the rate of pure time preference, the rate of risk aversion, and the growth rate of per capita consumption. The discussion on the rate of pure time preference has been voluminous (Nordhaus 2007;Pearce et al. 2003), but the rate of risk aversion attracted less attention in the context of climate change (Anthoff et al. 2009b;Anthoff et al. 2009c;Weitzman 2007). The per capita growth rate has been largely ignored in setting the appropriate discount rate for climate policy. This paper argues that it should not be.

Ramsey discounting is typically introduced as follows. To an individual, $100 in ten years time is worth less than $100 today because (1) we are impatient and (2) we expect to be richer in 10 years time. Income growth is evaluated at the margin by the product of the growth rate of monetary income and the curvature of the utility function. Income growth is thus transformed into utility growth. This measures how much we appreciate additional income. To a society, the arguments are the same, but then for an appropriately representative agent.

In the reasoning behind Ramsey discounting for individuals, money is hypothetically transferred from the present self to a future self. For societies, the transfer is from a representative agent at present to a representative agent in the future.[1] The question "who does this agent represent?" is ignored. Implicitly, the assessment assumes that the givers and receivers of the transfer are efficiently, equitably or randomly spread among the population.

This implicit assumption is completely wrong for climate policy. International agreements clearly state that the rich should take the lead in (paying for) greenhouse gas emission abatements (United Nations 1992). All empirical evidence has that the poor would suffer most from climate change and hence benefit most from mitigation (Tol 2009). Therefore, a hypothetical agent who represents mitigation effort is very different from an agent who represents climate change impacts (Schelling 1992;Schelling 2000). This paper considers the implications for Ramsey discounting and hence for climate policy.

The paper proceeds as follows. Section 2 sets up the definitions and relates to previous literature. Section 3 briefly presents the model. Section 4 discusses the numerical results that illustrate the impact of modified Ramsey discounting. Section 5 concludes.

2. Modified Ramsey rules

2.1. An ad hoc modification of the Ramsey rule

The Ramsey rule for the consumption discount rate is

$$(1) \qquad r_{j,t} = \rho + \eta g_{j,t}$$

where r is the discount rate of agent i at time t, ρ is the rate of pure time preference, η is the consumption elasticity of marginal utility, and g is the growth rate of income. Typically, agent j transfers money to a later self so that

$$(2) \qquad g_{j,t} = \frac{y_{j,t} - y_{j,t-1}}{y_{j,t-1}}$$

where y is income.

If agent j embarks on greenhouse gas emission reduction, she implicitly transfers money to other agents, who live later. Let us define a representative agent x, whose income equals

[1] Note that there is a tacit assumption that the transfer can be earmarked. This assumption is difficult to maintain between generations (Lind et al. 1998).

(3) $$y_{x,t} = \frac{\sum_r d_{r,t} y_{r,t}}{\sum_r d_{r,t}}$$

where *d* is the marginal damage suffered from climate change. That is, the income of the representative agent is the weighted average income of all agents, with the share in marginal damages as weights.

Then, the discount rate is

(4) $$\begin{aligned} r_{j,1} &= \rho + \eta \frac{y_{j,1} - y_{x,0}}{y_{x,0}} \\ r_{j,t} &= \rho + \eta g_{x,t}, t \geq 1 \end{aligned}$$

That is, in the initial period, the discount rate corrects for the income difference between the agent that undertakes abatement and the representative agent that receives the benefit. In later periods, the discount rate is driven by the income growth rate of the representative agent. Note that representative agent changes over time – see Equation (3).

4.2. A consistent modification of the Ramsey rule

Equations (3) and (4) are an intuitively clear but ad hoc modification of the Ramsey discount rate. However, the Ramsey discount rate is a result – not an assumption. The Ramsey discount rate follows from shifting consumption between periods while holding net present welfare constant. Deriving the Ramsey discount rate for the case of one investor and multiple beneficiaries, the discount rate for period 1 equals

(5) $$r_j(1) = \frac{(1+\rho)\sum_i d_{i,1}}{\sum_i \left[1 + \eta\left(y_{i,1} - y_{j,0}\right) y_{j,0}^{-1}\right]^{-1} d_{i,1}} - 1$$

Equation (5) is derived in the Appendix. It meets some basic checks: If there is only one actor, $d_{i,1}$ cancels and (5) reduces to (1). If η=0, r_j=ρ as in the original Ramsey rule.

For later periods, the discount rate equals

(6) $$r_j(t) = \frac{(1+\rho)\sum_i d_{i,t} \sum_i y_{i,t\text{-}1}^{-\eta} d_{i,t\text{-}1}}{\sum_i d_{i,t\text{-}1} \sum_i y_{i,t}^{-\eta} d_{i,t}} - 1$$

See below for further interpretation.

4.3. A differentiated, consistent modification of the Ramsey rule

The modified Ramsey rules use an agent who is representative for those who suffer the impacts of climate change. At the same time, the (modified) Ramsey discount rate varies over time and between alternative scenarios of economic growth. There is no reason why the discount rate should not also be differentiated between the victims of climate change.

If the investor in climate policy has a different discount rate for each person affected by climate change, then the standard Ramsey rule should be used between periods t and t+1, as there is no reason why the investor should use a different trade-off than the victim herself. The same principle is used in Equations (4) and (6), but for the representative victim.

As in Equations (4) and (5), a correction is made in the first period for the income difference between the investor and the victim. We show in the Appendix that the correction factor equals the ratio of the per capita incomes, raised to the power of the risk aversion, so that the social cost of carbon is calculated as:

$$(7) \qquad SCC^{REW}(j) = \sum_r \left(\frac{y_{j,0}}{y_{r,0}}\right)^{\eta} SCC_r = \sum_r \left(\frac{y_{j,0}}{y_{r,0}}\right)^{\eta} \sum_t \frac{d_{r,t}}{\prod_{s=1}^{t} 1+\rho+\eta g_{r,s}}$$

This is identical to regional equity weighting (Anthoff et al. 2009a). We use that term below because it has historical precedence and because "differentiated, consistent modified Ramsey discount rate" is an ugly mouthful.

4.4. Comparing the modifications

Equations (3-4) have an ad hoc modification of the Ramsey rule, while Equations (5-6) have a consistent modification. The two modifications are derived and presented differently. Rewriting (3-4) and reinserting the term $\rho\eta g$ – see Equation (A4) – we obtain

$$(8) \qquad r^a = \rho \frac{\eta \frac{\sum_i d_{i,t} y_{i,t}}{\sum_i d_{i,t}} + (1-\eta) \frac{\sum_i d_{i,t-1} y_{i,t-1}}{\sum_i d_{i,t-1}}}{\frac{\sum_i d_{i,t-1} y_{i,t-1}}{\sum_i d_{i,t-1}}} + \eta \frac{\left(\frac{\sum_i d_{i,t} y_{i,t}}{\sum_i d_{i,t}} - \frac{\sum_i d_{i,t-1} y_{i,t-1}}{\sum_i d_{i,t-1}}\right)}{\frac{\sum_i d_{i,t-1} y_{i,t-1}}{\sum_i d_{i,t-1}}}$$

Rewriting (5-6), we find

$$(9) \qquad r^c = \rho \frac{\sum_i d_{i,t} \sum_i y_{i,t\text{-}1}^{\text{-}\eta} d_{i,t\text{-}1}}{\sum_i d_{i,t\text{-}1} \sum_i y_{i,t}^{\text{-}\eta} d_{i,t}} + \frac{\sum_i d_{i,t} \sum_i y_{i,t\text{-}1}^{\text{-}\eta} d_{i,t\text{-}1} - \sum_i d_{i,t\text{-}1} \sum_i y_{i,t}^{\text{-}\eta} d_{i,t}}{\sum_i d_{i,t\text{-}1} \sum_i y_{i,t}^{\text{-}\eta} d_{i,t}}$$

Equations (8) and (9) are clearly different. The ad hoc modification uses the growth rate of the impact-weighted average of marginal utility. The consistent modification uses the impact-weighted average of the growth rate of marginal utility (recall that η>0), convoluted with the growth rate of the impacts.

Equation (4) has certain similarities to equity weighing. Equity weighing has that the net present globally aggregated impact is the weighted sum of the net present regional impacts. For a global planner (Fankhauser et al. 1997;Fankhauser et al. 1998), the weights are the ratio of the global average per capita income y_W over the regional average per capita income y_r, raised to the power of the risk aversion η:

$$(10) \qquad SCC^P = \sum_r \left(\frac{y_{W,0}}{y_{r,0}} \right)^\eta SCC_r$$

where SCC_r is the net present value of the marginal damage done by emitting an additional tonne of carbon dioxide:

$$(11) \qquad SCC_r = \sum_t \frac{d_{r,t}}{\prod_{s=1}^{t} 1 + \rho + \eta g_{r,s}}$$

For a regional planner j (Anthoff, Hepburn, and Tol 2009a), the ratio is per capita income of the abating region over the per capita income of the impacted region: Equation (7).

Similarly, Equation (4) multiplies the impacts in the first period with a weight that depends on relative per capita income:

$$(12) \qquad SCC^{AR}(j) = \sum_r W_j \sum_t \frac{d_{r,t}}{\prod_{s=1}^{t} 1 + \rho + \eta g_{x,s}}$$

where

$$(13) \qquad W_j = \frac{1 + \rho + \eta g_{x,t}}{1 + \rho + \eta \frac{y_{j,1} - y_{x,0}}{y_{x,0}}}$$

There are also differences, however. Firstly, the weight is different – as is immediately clear from comparing Equations (7) and (13). Equity weighing is driven solely by income differences between regions. Equation (4) is driven by income differences between polluters and pollutees. Secondly, the discount rate used is different. Equity weighing is based on region-specific growth rates, while the ad hoc modified Ramsey rule uses an average growth rate. Equity weights are static and determined by the current income distribution, while Equation (4) is different between scenarios of economic growth and impacts of climate change.

The social cost of carbon based on Equations (5) and (6) is different again. The weight in the first period is quite involved, and the discount rate is different.

3. The model

We use Version 2.9 of the Climate Framework for Uncertainty, Negotiation and Distribution (FUND). Version 2.9 of FUND has the same basic structure as that of Version 1.6 (Tol 1999;Tol 2001;Tol 2002c), except for the impact module (Tol 2002a;Tol 2002b). The source code and a complete description of the model can be found at http://www.fund-model.org/.

Essentially, FUND is a model that calculates damages of climate change and impacts of greenhouse gas emission reduction for 16 regions of the world by making use of exogenous scenarios of socioeconomic variables. The scenarios comprise of projected temporal profiles of population growth, economic growth, autonomous energy efficiency improvements and carbon efficiency improvements (decarbonization), emissions of carbon dioxide from land use change, and emissions of methane and of nitrous oxide. Carbon dioxide emissions from fossil fuel combustion are computed endogenously on the basis of the Kaya identity. The calculated impacts of climate change perturb the default paths of population and economic outputs corresponding to the exogenous scenarios. The model runs from 1950 to 2300 in time steps of a year, though the outputs for the 1950-2000 period is only used for calibration, and the years beyond 2100 are used for the approximating the social cost of carbon under low discount rates. The scenarios up to the year 2100 are based on the EMF14 Standardized Scenario, which lies somewhere in between IS92a and IS92f (Leggett et al. 1992). For the years from 2100 onward, the values are extrapolated from the pre-2100 scenarios. The radiative forcing of carbon dioxide and other greenhouse gases used by FUND is determined based on Shine *et al*. (1990). The global mean temperature is governed by a geometric buildup to its equilibrium (determined by the radiative forcing) with a half-life of 50 years. In the base case, the global mean temperature increases by 2.5˚C in equilibrium for a doubling of carbon dioxide equivalents. Regional temperature increases, which are the primary determinant of regional climate change damages (except for tropical cyclones, as discussed below), are calculated from the global mean temperature change multiplied by a regional fixed factor, whose set is estimated by averaging the spatial patterns of 14 GCMs (Mendelsohn et al. 2000).

The model considers the damage of climate change for the following categories: agriculture, forestry, water resources, sea level rise, energy consumption, unmanaged ecosystems, and human health (diarrhea, vector-borne diseases, and cardiovascular and respiratory disorders). Impacts of climate change can be attributed to either the rate of temperature change (benchmarked at 0.04˚C per year) or the level of temperature change (benchmarked at 1.0˚C). Damages associated with the rate of temperature change gradually fade because of adaptation (Tol 2002a).

People can die prematurely due to climate change, or they can migrate because of sea level rise. Like all impacts of climate change, these effects are monetized. The value of a statistical life is set to be 200 times the annual per capita income. The resulting value of a statistical life lies in the middle of the observed range of values in the literature (Cline 1992). The value of emigration is set to be 3 times the per capita income (Tol 1995), the value of immigration is 40 per cent of the per capita income in the host region (Cline 1992). Losses of dryland and wetlands due to sea level rise are modeled explicitly. The monetary value of a loss of one square kilometre of dryland was on average $4 million in OECD countries in 1990 (Fankhauser 1994). Dryland value is assumed to be proportional to GDP per square kilometre. Wetland losses are valued at $2 million per square kilometre on average in the OECD in 1990 (Fankhauser 1994). The wetland value is assumed to have logistic relation to per capita income. Coastal protection is based on cost-benefit analysis, including the value of additional wetland lost due to the construction of dikes and subsequent coastal squeeze.

Other impact categories, such as agriculture, forestry, energy, water, storm damage, and ecosystems, are directly expressed in monetary values without an intermediate layer of impacts measured in their ‘natural’ units (Tol 2002a). Impacts of climate change on energy consumption, agriculture, and cardiovascular and respiratory diseases explicitly recognize that there is a climatic optimum, which is determined by a variety of factors, including plant physiology and the behaviour of farmers. Impacts are positive or negative depending on whether the actual climate conditions are moving closer to or away from that optimum climate. Impacts are larger if the initial climate conditions are further away from the optimum climate. The optimum climate is of importance with regard to the potential impacts. The actual impacts lag behind the potential impacts, depending on the speed of adaptation. The impacts of not being fully adapted to new climate conditions are always negative (Tol 2002b).

The impacts of climate change on coastal zones, forestry, tropical and extratropical storm damage, unmanaged ecosystems, water resources, diarrhoea malaria, dengue fever, and schistosomiasis are modelled as simple power functions. Impacts are either negative or positive, and they do not change sign (Tol 2002b).

Vulnerability to climate change changes with population growth, economic growth, and technological progress. Some systems are expected to become more vulnerable, such as water resources (with population growth), heat-related disorders (with urbanization), and ecosystems and health (with higher per capita incomes). Other systems are projected to become less vulnerable, such as energy consumption (with technological progress), agriculture (with economic growth) and vector- and water-borne diseases (with improved health care) (Tol 2002b). The income elasticities (Tol 2002b) are estimated from cross-sectional data or taken from the literature.

4. Results

Table 1 shows the population and per capita income for the 16 regions in FUND for the year 2000, as well as the estimated social cost of carbon for emissions between 2000 and 2009, discounted to 2000. These numbers form the basis for the results below.

Table 2 shows the social cost of carbon for a pure rate of time preference of 1% per year and a consumption elasticity of marginal utility of unity. Results are shown for the five “OECD” “regions” of the FUND model: USA, Canada (CAN), Western Europe (WEU), Japan and South Korea (JPK) and Australia and New Zealand (ANZ). The first row has the regional social cost of carbon, which only considers the impacts on the own region. These numbers are small, as expected, and even negative for the Pacific regions. If Japan, South Korea, Australia, and New Zealand would care only about their own countries, they should subsidize greenhouse gas emissions.

Adding the regional social costs of carbon (of the five regions shown and the eleven not shown), the global social cost of carbon amounts of $7/tC. This does not account for income differences in the world. Using global equity weights, the social cost of carbon rises to $24/tC. The second row of Table 2 shows the results with regional equity weights (or differentiated, consistent modified Ramsey discount rates). Global equity weights assume a global social planner. To a first approximation, impacts are valued at the global average. Regional equity weights assume regional social planners. All impacts around the world are valued, to a first approximation, at the values of the region that reduces emissions. As a result, the regionally equity weighted social cost of carbon is much higher: between \$77/tC and \$176/tC. The estimates are ranked in the same order as the regional per capita income, with Australia and New Zealand at the bottom and Japan and South Korea at the top.

The third row of Table 2 shows the social cost of carbon using the ad hoc modification of the Ramsey rule, and the fourth row contains the results for the consistent modification (not differentiated between regions).

With the ad hoc modification, the social cost of carbon is lower than with regional equity weighing, but the regional results are in the same order: The poorest of the rich regions is at the bottom, and the richest at the top. This follows from Equation (4): The larger the difference in income between the evaluating region and the representative region for impacts, the larger the weight in the first period. The economic growth rate used for later periods is independent of the evaluating region – see Equation (4).

With the consistent modification of the Ramsey rule, the social cost of carbon is numerically similar to the regional-equity-weighted estimates. Estimates may be slightly higher or slightly lower. The order is the same, and follows from Equation (5). The initial correction is a weighted average of the income difference in the first period between the evaluating regions and the affected regions. The growth rate used for later periods – see Equation (6) – is the same for all regions.

Figure 1 illustrates the mechanisms behind the estimates of the social cost of carbon for the USA. The incremental impact is initially negative – that is, additional warming is good in the short term – but it rapidly turns positive and grows to some seven cents around 2150 and then levels off as the pulse of carbon dioxide is removed from the atmosphere. If the standard Ramsey rule is applied without equity weighing, the incremental impact is muted, particularly in the long term. With global or regional equity weights, the pattern is the same over time but multiplied by the initial weight. The ad hoc and consistent modified Ramsey rules apply both a different initial weight and use a different discount rate. Figure 1 shows that regional equity weighing and consistent modified Ramsey rule follow roughly the same pattern (which explains why the estimates of the social cost of carbon are so close) but a different pattern nonetheless.

Figure 2 shows the initial weights applied by the USA to the impacts on all regions, averaged over those regions for a rate of risk aversion of unity. The order of the weights is different than the order of the social cost of carbon. The simple sum gives the lowest at unity (by definition); the social cost of carbon is lowest. However, the global (regional) equity weight gives a higher initial weight than the ad hoc (consistent) modified Ramsey rule but the social cost of carbon is lower. That means that the discount factor over the entire period is lower.

Table 3 repeats Table 2 for a pure rate of time preference of 3% per year instead of 1%. Obviously, the estimates of the social cost of carbon are much lower than for a pure rate of time preference of 1%. The pattern of Table 2 is largely preserved, however. The richest regions have the highest social costs of carbon. The ad hoc modified Ramsey rule has the lowest social costs of carbon. The results consistent modified Ramsey rule are now always lower than those for the regional equity weighting; and the numerical difference between the two is much larger in a relative sense.

Table 4 repeats Table 2 for a rate of risk aversion of 2 instead of 1. The rate of risk aversion has a double role. It determines the weight placed on the income differences between the evaluating region and the impacted regions. Here, the social cost of carbon increases with the rate of risk aversion. The rate of risk aversion also features in the discount rate. Here, the social cost of carbon falls with the rate of risk aversion. The equity effect dominates the discounting effect in all cases. The pattern of Tables 2 and 3 is repeated in Table 4: Richer regions have a higher social cost of carbon.

5. Discussion and conclusion

In this paper, we introduce three modifications of the Ramsey discount rate for use in evaluating greenhouse gas emission reduction. While the original Ramsey rule was designed to evaluate transfers from a current to a future self, the modified Ramsey rules evaluate transfers from a current self, who bears the cost emission abatement, to a future other, who is representative for the beneficiaries of emission reduction. The modified Ramsey rules have two components. In the first period, there is a correction for the income difference of the self and the other. In later periods, the income growth rate of the other is used, corrected for shifts in the representativeness of the other. We propose three modifications. The first is ad hoc yet intuitive, the other two are consistent with the welfare theory underlying the original Ramsey rule. The consistent modified Ramsey rules either differentiate between regions or use a weighted average. The differentiated, consistent modified Ramsey rule is identical to the regional equity weights proposed earlier by (Anthoff, Hepburn, and Tol 2009a).

We apply the modified Ramsey rules to estimates of the social cost of carbon. The modified Ramsey rules lead to substantially higher cost estimates. The ad hoc modification implies lower estimates than the consistent modified Ramsey rules. The results of the (undifferentiated) consistent modification are numerically close to, although generally somewhat lower than estimates using the standard Ramsey rule plus regional equity weights. The estimated social costs of carbon differ between regions, with richer regions facing higher cost estimates. This confirms that optimal climate policy has different carbon tax rates for different countries (Chichilnisky et al. 1994;Sheeran 2006).

These results underline the importance of discounting and the distribution of income and impacts for evaluating the appropriate level of a carbon tax. Note that the framework presented here is incomplete. We ignored distributional issues within regions, and uncertainty. We assumed that the trade-offs between people living at the same time are governed by the same curvature of the utility function as trade-offs between people living at different times. We ignored that population growth is endogenous. These issues are deferred to future research.

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Appendix: The Ramsey rate of consumption discount

A.1. The standard Ramsey rule

Let us consider one person, two periods. The welfare function is:

(A1) $$W = U(C_0) + \frac{U(C_1)}{1+\rho}$$

where *W* is the net present value of welfare, *U* is the utility function, *C* is consumption at times 0 and 1, and ρ is the rate of pure time preference or the utility discount rate.

The consumption discount rate is defined as

(A2) $$r := \frac{\mathrm{d}C_1}{\mathrm{d}C_0} - 1$$

under the condition that the consumption discount rate preserves net present welfare:

(A3) $$\mathrm{d}W = U_{C_0}\mathrm{d}C_0 - \frac{U_{C_1}\mathrm{d}C_1}{1+\rho} = 0$$

Combining (2) and (3) implies

(A4) $$r = \frac{(1+\rho)U_{C_0} - U_{C_1}}{U_{C_1}} \approx \frac{(1+\rho)U_{C_0} - \frac{1}{1+\eta g}U_{C_0}}{\frac{1}{1+\eta g}U_{C_0}}$$

$$(1+\rho)(1+\eta g) - 1 \approx \rho + \eta g$$

A.2. A consistent modification of the Ramsey rule: Single discount rate

Now consider a social welfare problem:

(A5) $$W = \sum_i U(C_{i,0}) + \sum_i \frac{U(C_{i,1})}{1+\rho}$$

where *W* is the net present value of welfare, *U* is the utility function, *C* is consumption at times 0 and 1, and ρ is the rate of pure time preference or the utility discount rate.

For the first period, the consumption discount rate for an investment by agent *j* is defined as

(A6) $$r_j(0) := \frac{\sum_i \mathrm{d}C_{i,1}}{\mathrm{d}C_{j,0}} - 1$$

under the condition that the consumption discount rate preserves net present welfare:

(A7) $$\mathrm{d}W = U_{C_{j,0}}\mathrm{d}C_{j,0} - \sum_i \frac{U_{C_{i,1}}\mathrm{d}C_{i,1}}{1+\rho} = 0$$

Combining (A6) and (A7) implies

$$r_j(0) = \frac{(1+\rho)U_{C_{j,0}} \sum_i \mathrm{d}C_{i,1} - \sum_i U_{C_{i,1}} \mathrm{d}C_{i,1}}{\sum_i U_{C_{i,1}} \mathrm{d}C_{i,1}} \approx$$

$$\text{(A8)} \quad \frac{(1+\rho)U_{C_{j,0}} \sum_i \mathrm{d}C_{i,1} - \sum_i \frac{1}{1+\eta g_i} U_{C_{j,0}} \mathrm{d}C_{i,1}}{\sum_i \frac{1}{1+\eta g_i} U_{C_{j,0}} \mathrm{d}C_{i,1}}$$

$$= \frac{(1+\rho)\sum_i \mathrm{d}C_{i,1}}{\sum_i \frac{1}{1+\eta g_i} \mathrm{d}C_{i,1}} - 1 = \frac{(1+\rho)\sum_i \mathrm{d}C_{i,1}}{\sum_i \frac{1}{1+\eta\left(y_{i,1} - y_{j,0}\right) y_{j,0_i}^{-1}} \mathrm{d}C_{i,1}} - 1$$

Note that the approximation between the first and second line introduces a singularity. The approximation is used for interpretation only; all numerical results are based on the first line of (A8).

For later periods, the discount rate is defined as

$$\text{(A9)} \quad r := \frac{\sum_i \mathrm{d}C_{i,t+1}}{\sum_i \mathrm{d}C_{i,t}} - 1$$

under the condition that the consumption discount rate preserves net present welfare:

$$\text{(A10)} \quad \mathrm{d}W = \sum_i U_{C_{i,t}} \mathrm{d}C_{i,t} - \sum_i \frac{U_{C_{i,t+1}} \mathrm{d}C_{i,t+1}}{1+\rho} = \sum_i \mathrm{d}C_{i,t} \frac{\sum_i U_{C_{i,t}} \mathrm{d}C_{i,t}}{\sum_i \mathrm{d}C_{i,t}} - \sum_i \frac{U_{C_{i,t+1}} \mathrm{d}C_{i,t+1}}{1+\rho} = 0$$

Combining (A9) and (A10), we find

$$\text{(A11)} \quad r_j(t) = \frac{(1+\rho)\sum_i \mathrm{d}C_{i,t+1} \sum_i U_{C_{i,t}} \mathrm{d}C_{i,t} - \sum_i \mathrm{d}C_{i,t} \sum_i U_{C_{i,t+1}} \mathrm{d}C_{i,t+1}}{\sum_i \mathrm{d}C_{i,t} \sum_i U_{C_{i,t+1}} \mathrm{d}C_{i,t+1}}$$

Unlike (A8), this expression cannot be further simplified.

A.2. A consistent modification of the Ramsey rule: Multiple discount rates

Again consider a social welfare problem (A5). For the first period, the consumption discount rate for an investment by agent *j* for the benefit of agent *i* is defined as

$$\text{(A12)} \quad r_{ji}(0) := \frac{\mathrm{d}C_{i,1}}{\mathrm{d}C_{j,0}} - 1$$

under the condition that the consumption discount rate preserves net present welfare:

(A13) $$\mathrm{d}W = U_{C_{j,0}}\mathrm{d}C_{j,0} - \frac{U_{C_{i,1}}\mathrm{d}C_{i,1}}{1+\rho} = 0$$

Combining (A12) and (A13) implies

(A14) $$r_{ji}(0) = \frac{(1+\rho)U_{C_{j,0}} - U_{C_{i,1}}}{U_{C_{i,1}}} = \frac{(1+\rho)U_{C_{j,0}} - \dfrac{U_{C_{i,0}}}{1+\eta g_i}}{\dfrac{U_{C_{i,0}}}{1+\eta g_i}} =$$
$$(1+\rho)(1+\eta g_i)\frac{U_{C_{j,0}}}{U_{C_{i,0}}} - 1 \approx (1+\rho+\eta g_i)\frac{U_{C_{j,0}}}{U_{C_{i,0}}} - 1$$

The discount factor then is

(A15) $$DF_{ji}(0) := \frac{1}{1+r_{ji}(0)} = \frac{U_{C_{i,0}}}{U_{C_{j,0}}}\frac{1}{1+\rho+\eta g_i}$$

For later periods, the discount rate is defined as

(A16) $$r := \frac{\mathrm{d}C_{i,t+1}}{\mathrm{d}C_{i,t}} - 1$$

under the condition that the consumption discount rate preserves net present welfare:

(A17) $$\mathrm{d}W = U_{C_{i,t}}\mathrm{d}C_{i,t} - \frac{U_{C_{i,t+1}}\mathrm{d}C_{i,t+1}}{1+\rho}$$

Combining (A16) and (A17), we find

(A18) $$r_{ji}(t) \approx \rho + \eta g_i$$

That is, if the investor in emission reduction has a different consumption discount rate for each victim of climate change, then the investor would use the standard Ramsey rate of discount (with the growth rate of the victim) – see Equation (A18) – but multiply the net present value of the impact with the ratio of marginal utility (A15). This is identical to regional equity weighting.

Table 1. Population and per capita income in 2000, and the estimated regional social cost of carbon for different rates of risk aversion (η) and pure time preference (ρ).

	Population	Income	Social cost of carbon ($/tC)		
	mln	$/p/yr	η=1, ρ=0.01	η=1, ρ=0.03	H=2, ρ=0.01
USA	278	37317	2.36	0.38	0.67
Canada	31	25927	0.14	0.01	0.03
Western Europe	388	32417	4.71	0.60	1.22
Japan and South Korea	171	49762	-0.80	-1.03	-0.96
Australia and New Zealand	20	21694	0.00	-0.06	-0.04
Eastern Europe	125	3235	0.15	-0.02	-0.02
former Soviet Union	293	2146	1.35	0.30	0.21
Middle East	241	2524	0.12	-0.22	-0.23
Central America	137	2830	0.11	-0.08	-0.09
South America	346	3901	0.32	0.00	-0.02
South Asia	1365	607	0.63	-0.07	-0.09
Southeast Asia	615	1775	1.95	0.33	0.21
China, North Korea and Mongolia	1313	1778	4.15	-0.04	-0.30
North Africa	143	1491	1.12	0.29	0.26
Sub-Saharan Africa	635	476	0.73	0.15	0.13
Small Island States	43	1259	0.07	-0.02	-0.02

Table 2. The social cost of carbon ($/tC) for five regions of the OECD, for a pure rate of time preference of 1% per year and a rate of the risk aversion of 1.

	USA	CAN	WEU	JPK	ANZ
Regional	2.36	0.14	4.71	-0.80	0.00
Equity weighted[a]	293.48	203.91	254.95	391.37	170.62
Ad hoc mod. Ramsey	159.45	110.78	138.51	212.62	92.70
Consistent mod. Ramsey	297.47	205.70	255.98	388.30	170.47
Simple sum[b]	17.12	Equity weighted[c]			52.38

[a] Equity weights with regional normalization.

[a] Global social cost of carbon without equity weights.

[c] Global social cost of carbon with equity weights.

Table 3. The social cost of carbon ($/tC) for five regions of the OECD, for a pure rate of time preference of 3% per year and a rate of the risk aversion of 1.

	USA	CAN	WEU	JPK	ANZ
Regional	0.38	0.01	0.60	-1.03	-0.06
Equity weighted[a]	21.64	15.04	18.80	28.86	12.58
Ad hoc mod. Ramsey	17.05	11.85	14.82	22.74	9.91
Consistent mod. Ramsey	19.53	13.50	16.81	25.49	11.19
Simple sum[b]	0.53	Equity weighted[c]			3.86

[a] Equity weights with regional normalization.

[a] Global social cost of carbon without equity weights.

[c] Global social cost of carbon with equity weights.

Table 4. The social cost of carbon ($/tC) for five regions of the OECD, for a pure rate of time preference of 1% per year and a rate of the risk aversion of 2.

	USA	CAN	WEU	JPK	ANZ
Regional	0.67	0.03	1.22	-0.96	-0.04
Equity weighted[a]	558.56	269.64	421.51	993.28	188.78
Ad hoc mod. Ramsey	186.97	90.26	141.10	332.49	63.19
Consistent mod. Ramsey	489.28	233.97	362.31	833.69	160.69
Simple sum[b]	0.96	Equity weighted[c]			17.79

[a] Equity weights with regional normalization.

[a] Global social cost of carbon without equity weights.

[c] Global social cost of carbon with equity weights.

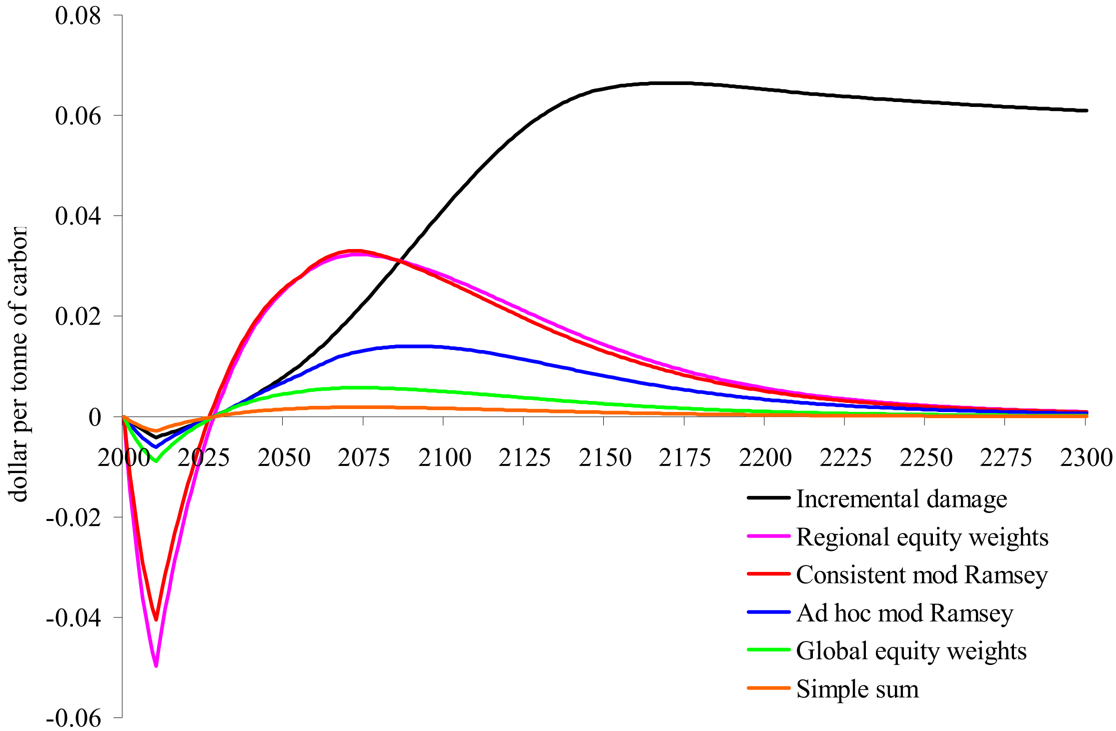

Figure 1. The current and present values of the normalized incremental damage due to increased carbon dioxide emissions in 2005; present values are shown for the standard Ramsey rule of discounting without equity weighing (simple sum) and with regional (USA) and global equity weighing, as well as for the ad hoc modified Ramsey rule and the consistent modified Ramsey rule; the pure rate of time preference is 1% per year; the rate of risk aversion is unity.

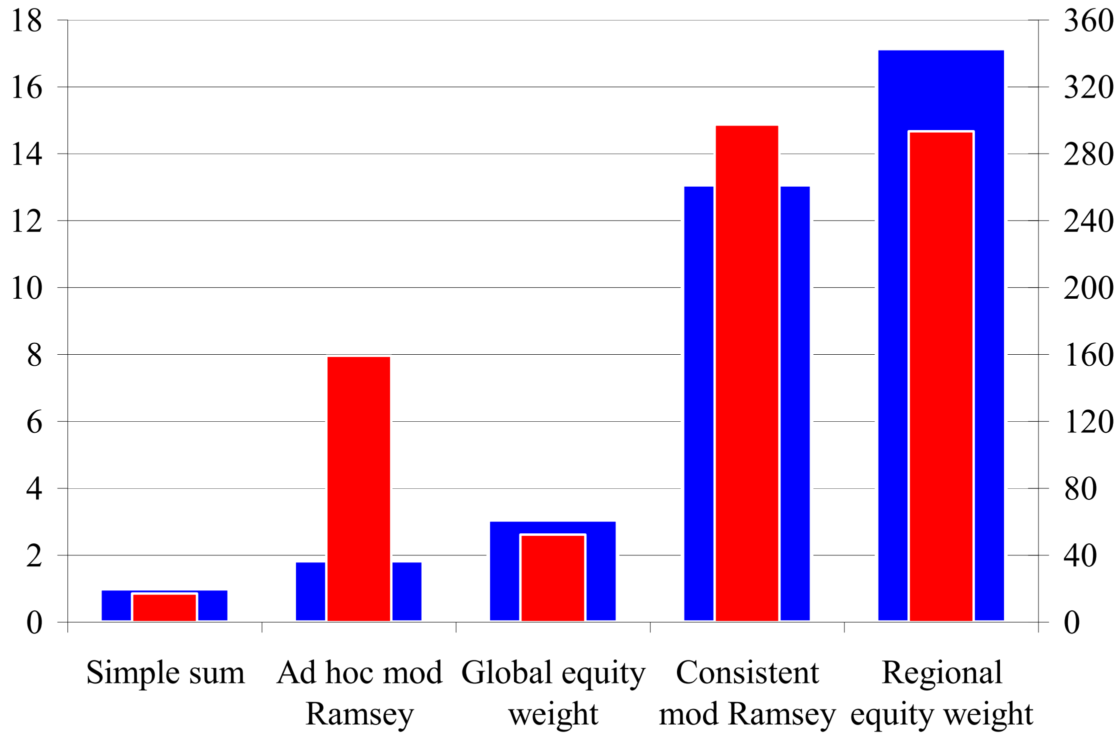

Figure 2. The social cost of carbon (red, narrow bar; right axis) and the initial weight (blue, wide bar; left axis) applied to the incremental damage due to increased carbon dioxide emissions in 2005 for the standard Ramsey rule without equity weighing (simple sum) and with regional and global equity weighing, as well as for the ad hoc modified Ramsey rule and the consistent modified Ramsey rule; the rate of risk aversion is unity.

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