Optimal Retirement and Saving with Healthy Aging

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Abstract

We construct a model in which retirement occurs at the end of life because of declining health. We show that, with complete markets, improvements in life expectancy, coupled with a delay in the onset of disability, create a wealth effect that increases both the optimal consumption level and proportion of life spent in leisure. Social security systems that wish to mimic optimal behavior should respond to longer, healthier, lives by reducing contribution rates and increasing benefit rates, funded by an increase the retirement age that can be less than proportional to the rise in adult life expectancy.

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1. Introduction

The improvement in life expectancy and living standards over the last 150 years constitutes a dramatic increase in human welfare. For the world as a whole, life expectancy at birth rose from around 30 years in 1900 to 65 years by 2000 (and is projected to rise to 81 by the end of this century; Lee 2003). These improvements have not only resulted in a large direct gain in welfare (Nordhaus 2003, Becker, Philipson, and Soares, 2005) but have also had a profound influence on economic life-cycle behavior by changing people's time horizons (Hamermesh, 1985).

In this paper we extend the Blanchard-Yaari-Weil model of consumption with finite lives and perfect annuity markets to endogenously determine retirement, as well as the time path of consumption. We use the model to show how optimal retirement and saving decisions respond to changes in the level of wages and life expectancy. A major difficulty for an economic explanation of retirement is the lumping of leisure at the end of the life whereas standard concavity conditions would usually imply the smoothing of leisure over time. We explain retirement by rising ill health with age that increases the disutility of labor and reduces the productivity of working.

While health declines with age, rising life spans have been accompanied by improving age-specific health status; that is, we have lives that are both longer and healthier. The ‘compression of morbidity’ hypothesis (Fries, 1980), maintains that the average age at first infirmity, disability, or other morbidity is postponed to such an extent that the period of ill health at the end of life is compressed. Increases in life expectancy in the United States over the last two centuries have indeed been associated with reductions in the age-specific incidence of disease, disability, and morbidity (Costa 2002; Fogel 1994, 1997). This trend is continuing (Crimmins, Saito and Ingegneri, 1997, Freedman, Martin and Schoeni 2002). We model the compression of morbidity by assuming that the age of onset of disability rises proportionately, or more than proportionately, with life expectancy.

Increases in both wages and lifespan have wealth effects in that they enlarge the budget set. They also have incentive and substitution effects by changing the rewards to working and saving. Our model shows that, under standard assumptions on preferences, the wealth effects tend to dominate, producing increased consumption and leisure when wages and life expectancy increase. The wealth effect of an increase in life expectancy will reduce the proportion of lifespan devoted to work but is unlikely to reduce the absolute length of the working life.
On the other hand we show that increases in the level of wages will reduce the length of the working life if the workers intertemporal elasticity of substitution with respect to consumption is less than one (corresponding to a coefficient of relative risk aversion greater than one).

We use these results to argue that the long-term decline in the age of retirement in industrial countries over the last 150 years, the labor force participation rate of men in the United States aged 65 and older declined from around 80 percent in 1850 to less than 20 percent by 2000 (Costa, 1998b), has been due to the increasing *level* of wages as a driving force behind earlier retirement, which coincides with Costa's (1998a) analysis.

As well as offering a positive theory for observed behavior, our approach offers a method to estimate the welfare effects of proposed changes to social security systems. Insofar as social security systems try to promote the optimal retirement and consumption outcomes rational agents would make with complete markets, the results of our model can serve as benchmarks in the design of public pension systems. Our model implies that consumption should rise when life expectancy increases.

A seemingly natural approach to dealing with solvency problems that arise in a social security system when life expectancy increases is to increase contribution rates and reduce benefit rates. Our model implies that the optimal response should be to reduce contribution rates and increase benefit rates, maintaining solvency exclusively by increasing the age of retirement, though it can rise less than proportionately with life expectancy. The reason that this plan can maintain solvency is that rising wages over time and compound interest on accumulated savings means longer working lives tend to create more than proportionately higher wealth at retirement. We show that under standard assumptions on interest rates and wage growth such a plan is feasible given projected changes in age specific survival curves.

Several papers have tackled the issue of how changes in longevity affect retirement and consumption decisions. Chang (1991) argues that with compete markets a rise in life expectancy reduces consumption and increases savings rates. The major difference between his model and ours is that he assumes that when life expectancy rises the forces that give rise to retirement at each age (low wages and a high disutility of labor) remain unchanged, whereas we assume healthy life expectancy also rises, making a longer working life more palatable. Chang (1991) and Kalemli-Ozcan and Weil (2002) also emphasize that with no annuity markets a reduction in mortality rates increases the effective return to saving and encourages saving and early retirement when life expectancy rises.

Our approach differs in that we emphasize that wealth effect, leading to increased consumption (reduced saving) and more leisure with longer life spans, tends to dominate when markets are complete. Bloom, Canning, Mansfield, and Moore (2007) present a simple model of retirement and saving with complete markets, assuming that utility is logarithmic in consumption and that the mortality rate is constant, independent of age. We generalize the results of that paper by allowing a general concave utility function, and varying age specific mortality rates.

We present our model in section 2, and in section 3 we show how the dynamic programming problem facing agents generates a set of equations (derived from the first-order conditions) that determine the optimal retirement age and consumption profile. In section 4 we use the model to investigate the effect of increases in healthy lifespan and wages on savings and retirement behavior and give our results. Section 5 discusses the implications of our results for social security systems and section 6 makes some concluding remarks.

2. The Model

This life-cycle theory of saving and retirement is not a complete model of saving and retirement behavior. For example, people also engage in precautionary savings to guard against income shocks, and they may also save to provide bequests (Skinner 1985). In addition, in many developing countries the elderly are supported to a large extent by intra-family transfers, while many industrial countries have widespread social security systems that support the elderly. These transfer systems clearly affect incentives to save and to retire. Notwithstanding its limitations, the life-cycle model can provide valuable insights into behavior. In addition, optimal decisions by individuals with complete markets generates an efficient outcome which can be used to guide policy even under market imperfections.

Our formal lifecycle model makes a number of simplifying assumptions. We assume that the mortality schedule is exogenous, ignoring the possibility of using consumption and health services to extend longevity (Ehrlich and Chuma 1990), or of a reverse link from labor supply to health status and life expectancy. In the interest of simplicity, we also assume that the life cycle has no period of schooling. Longer life spans may increase the incentive to invest in education (De la Croix and Licandro 1999) though it is really only duration of working life that earns a return to education (Echevarría and Iza 2006) and the education effect depends on the effect of life span on retirement.

We begin our formal model with mortality. We assume that there is a family of possible survival schedules indexed by a single variable λ. There is a survival schedule $s(t,\lambda)$ that gives the probability of survival to age t. The mortality rate at time t is

$$m(t,\lambda) = -\frac{\frac{ds}{dt}(t,\lambda)}{s(t,\lambda)} \qquad (1)$$

and life expectancy is given by[1]

$$z(\lambda) = \int_0^T t\, m(t,\lambda)\, s(t,\lambda) dt = \int_0^T s(t,\lambda) dt \qquad (2)$$

We assume that there is a biological maximum to the length of life (Carnes, Olshansky, and Grahn, 2003). Let us assume that as the index λ increases, mortality at each age falls, so that survival to each age rises, as does overall life expectancy, z, and that this relationship is invertible. Lee and Miller (2001) show using empirical data that in practice the Lee and Carter (1992) method of indexing mortality rates generates a monotonic relationship between the index and the age-specific mortality rates; adopting this method of indexing would therefore imply an invertible mapping between the parameter λ and life expectancy. Assuming a one-to-one mapping from life expectancy to our index of mortality, we can take life expectancy to be a unique identifying index for a mortality schedule. In what follows we therefore use life

[1] The probability that your age of death is exactly t is the probability of surviving to t times the mortality rate at t. The second equality comes from substituting in for the mortality rate and integrating by parts using the fact that $s(0,\lambda)=1$ and $s(T,\lambda)=0$.

expectancy at the beginning of working life, z, as our index of the mortality schedule and write mortality[2] as $m(t,\ z)$

A major innovative feature of our model is the health schedule $h(t,z)$. We assume that health declines with age t. In addition, we postulate that health at age t depends on life expectancy, z. If health is independent of z, it implies that increases in life expectancy are not associated with general health improvements in the form of reductions in morbidity (sickness), which implies further that population aging is associated with an increasingly large share of unhealthy people.

However the evidence points in the opposite direction. Not only are people living longer; age specific disability is falling. Cutler (2001) attributes these reductions in disability at older ages to the long term protective effects of reduced disease exposure in childhood health, rising levels of education and socio-economic status, improved health related behavior such as reductions in smoking, improved medical care, and the use of special aids to reduce disability for a given health state. Manton, Gu, and Lamb (2006) estimate that between 1935 and 1982 life expectancy and (disability free) active life expectancy at age 65 moved up together in the United States, keeping the proportion of life after 65 that was disability free relatively steady at around 73%. However between 1982 and 1999 life expectancy at age 65 rose from 16.9 to 17.7 years while active life expectancy rose from 12.3 to 13.9 years, implying a rise in proportion of disability free life to over 78%, and a reduction in the absolute time spent in disability from 4.6 to 3.8 years. The compression of morbidity, either absolute or relative, appears to hold in most developed countries (Mor, 2005) and while time series evidence in developing countries is scarce, Mathers et al. (2001) show that across countries health-adjusted life expectancy (each life

[2] Note that we do not impose the Lee-Carter functional form; we only specify that there is a one-to-one mapping from mortality schedules to life expectancy so that life expectancy identifies a mortality schedule.

year weighted by a measure of health status) increases as fast as total life expectancy, implying a rising proportion of healthy to total life expectancy as life expectancy increases..

While the compression of disability, at least in the recent past, in the United States seems clear, and disability is a cause of retirement (Gordo, 2006), most people retire before the onset of severe disability; while they may still be physically capable of working, moderate ill health that does not amount to disability, may deter labor supply. The “dynamic equilibrium” hypothesis suggests that while time spent in serve disability is being compressed, this compression may be the result of reducing the transition rate from low or moderate ill health to severe disability resulting in an increase in the prevalence of less severe disability states (e.g. Graham et al. 2004).

To examine the effect of the compression of morbidity we begin by taking as our bench mark the case where healthy life span increases proportionately with overall lifespan. This is we assume:

$$h(\rho t, \rho z) = h(t, z) \;\; for \;\; \rho > 0 \qquad (3)$$

Note that we assume both age and life expectancy are measured form the start of adult life. If a particular level of health is labeled “disabled” equation (3) implies that the age of onset of disability rises proportionately with life expectancy. After examining this case we discuss how our results extend to the case

$$h(\rho t, \rho z) > h(t, z) \;\; for \;\; \rho > 1 \qquad (4)$$

which allows for the absolute compression of morbidity.

We denote labor supply at time t by χ_t , which is assumed to lie in the closed interval $[0,1]$. Felicity is assumed to be strongly separable in goods and leisure and is given by

$$u(c(t)) - \chi(t) v(h(t, z)) \qquad (5)$$

where $c(t)$ is consumption at age t and $v(h(t, z))$ is the disutility of working given the health state $h(t, z)$. We assume that u is twice differentiable with $u'(c) > 0, u''(c) < 0$ and the disutility function v satisfies $v'(h) < 0$ so that the disutility of work (and the relative utility of leisure) is higher when health is poorer. Note that in this respect health human capital may be very different from education human capital: Heckman (1976) assumes that the utility of leisure increases with education. Lifetime expected utility is

$$U = \int_0^T e^{-\delta t} s(t,z)\left[u(c(t)) - \chi(t)v(h(t,z))\right]dt \qquad (6)$$

Where δ is the subjective rate of time preference, and T is the biological maximum lifespan. The wage earned by a worker with health h is given by

$$w(t,h) = w_1(t)w_2(h(t,z)) \qquad (7)$$

The term $w_1(t)$ captures the change in wages over time due to exogenous forces, for example, technical progress and $w_2(h(t,z))$ captures the fact that worker productivity depends on health, which is a function of age relative to life expectancy. We assume $w_1'(t) \geq 0$ and $w_2'(h) \geq 0$ so that wages are increasing over time due to exogenous forces and are increasing in health. The multiplicative functional form implies that the health effect on log wages is additive, which is consistent with a Mincer wage equation that includes health as a form of human capital as in Schultz (1999). Chang (1991) and Kalemli-Ozcan and Weil (2002) have exogenous age-wage schedules that do not change as life expectancy varies; we allow for an effect of life expectancy on wages through improved health.

Wealth, W, evolves according to

$$\frac{dW}{dt} = \chi(t)w_1(t)w_2(h(t,z)) + \left(m(t,z)+r\right)W(t) - c(t) \qquad (8)$$

If the agent works at time t, he or she earns the wage $w_1(t)w_2(h(t,z))$, and is added to wealth, while consumption, $c(t)$, reduces wealth, while the stock of wealth earns a return $m(t,z)+r$.

We assume that health may affect labor productivity and wages. We assume that wealth can be transferred from one period to another by saving or borrowing from the financial sector. This competitive financial sector can borrow or lend freely at the interest rate *r*. Agents, however, are paid an effective interest rate $m(t,z)+r$ on their savings, which is larger than *r,* to compensate them for the fact that they may die before withdrawing their savings. Similarly, agents who borrow pay the rate $m(t,z)+r$ to compensate the bank for the fact that they may die before repaying their borrowings. This is equivalent to treating all savings as being in the form of annuity purchases, while all borrowing has to be accompanied by an actuarially fair life insurance contract for the amount of the loan. Provided that a continuum of agents exists, the financial sector can avoid all risk by aggregating over individuals and earn zero profits.

The transfer of the wealth of those who die to the financial sector exactly compensates deposit-taking institutions for the fact that they pay an interest rate $m(t,z)+r$ on deposits that exceed the risk-free rate *r*, and rules out the need to consider unintended bequests. The budget constraint is:

$$\int_0^T e^{-rt} s(t,z) c(t) dt \le \int_0^T e^{-rt} s(t,z) \chi(t) w_1(t) w_2(h(t,z)) dt \tag{9}$$

The control variables for the agent's optimization problem are c and χ. Agents must decide when to work and when to retire and what their consumption stream should be[3].

We assume the boundary conditions:

$$\begin{aligned} w_1(0) w_2(h(0,z)) u'(0) &> v(h(0,z)) \\ w_1(T) w_2(h(T,z)) u'(0) &< v(h(T,z)) \end{aligned} \tag{10}$$

The first boundary condition implies that the disutility of working at age zero is sufficiently low to ensure that the agent always works at the start of life, rather than live with no consumption.

[3] Adding the direct utility of health as an additive term to the utility function does not affect decision making in any way.

The second boundary condition implies that the disutility of working at the maximum possible age is sufficiently high that the agent always prefers to be retired.

It what follows we will assume that the rate of interest, r, is positive and equals the rate of time preference, δ. Our theory is developed for the case where the growth rate of the exogenous component of wages is moderate. If wage growth is very rapid we can have a number of unusual outcomes. The first is that if wage growth is very rapid a retired worker may renter the labor market to take advantage of the high wages when old. To rule this out we assume that for every life expectancy, the rate of growth of the disutility of labor with age exceeds the exogenous component in the rate of growth of wages at each point in time:

$$\frac{\dot{w}_1}{w_1} < \frac{\dot{v}}{v} \tag{11}$$

This condition will imply that retirement occurs only once and is never reversed. We can label the retirement age in this case as R.

3. Optimal Retirement and Consumption Decisions

The Hamiltonian for this problem is

$$\begin{aligned} H &= e^{-\delta t} s(t,z)\left[u(c(t)) - \chi(t)v(h(t,z))\right] \\ &\quad + \phi(t)[\chi(t)w_1(t)w_2(h(t,z)) + \left(m(t,z)+r\right)W(t) - c(t)] \end{aligned} \tag{12}$$

The first-order conditions for a maximum in c and χ are:

$$\dot{\phi} = -\frac{\partial H}{\partial W} = -\phi(t)(r + m(t,z)), \tag{13}$$

$$\frac{\partial H}{\partial c} = e^{-\delta t} s(t,z) u'\left(c(t)\right) - \phi(t) = 0, \text{ and} \tag{14}$$

$$\frac{\partial H}{\partial \chi} = -e^{-\delta t} s(t,z) v(h(t,z)) + \phi(t) w_1(t) w_2(h(t,z)) \geq 0 \text{ when } \chi(t) = 1$$
$$\frac{\partial H}{\partial \chi} = -e^{-\delta t} s(t,z) v(h(t,z)) + \phi(t) w_1(t) w_2(h(t,z)) = 0 \text{ when } \chi(t) \in (0,1) \qquad (15)$$
$$\frac{\partial H}{\partial \chi} = -e^{-\delta t} s(t,z) v(h(t,z)) + \phi(t) w_1(t) w_2(h(t,z)) \leq 0 \text{ when } \chi(t) = 0$$

These conditions can be shown to yield the following[4]:

$$\frac{\dot{c}}{c} = (r - \delta) \frac{u'(c)}{-cu''(c)} \qquad (16)$$

$$u'(c(t)) w_1(t) w_2(h(t,z)) > v(h(t,z)) \Rightarrow \chi(t) = 1$$
$$u'(c(t)) w_1(t) w_2(h(t,z)) = v(h(t,z)) \Rightarrow \chi(t) \in [0,1] \qquad (17)$$
$$u'(c(t)) w_1(t) w_2(h(t,z)) < v(h(t,z)) \Rightarrow \chi(t) = 0$$

.

The first condition is standard and arises because the annuities perfectly insure against financial losses due to death. Given our assumption that $r = \delta$, optimal consumption is constant over time. The second condition implies that agents work at time t so long as the utility gain from the consumption purchased by the wage they earn (the marginal utility of consumption times the wage) exceeds the disutility of working.

The concavity of the utility function u implies that the Hamiltonian is concave in the two control variables. Hence these first-order conditions give a global maximum of the Hamiltonian in the control variables, c and χ. The Hamiltonian is jointly concave[5] in the two control variables and the state variable, wealth, and hence satisfies the Mangasarian condition for our set of equations to give a maximum for the dynamic programming problem (see Sydsaeter, Storm and Berck, 2000).

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[4] Equation (16) follows from equation (1). We suppress the fact that consumption depends on t though this is implicit in equation (16).
[5] The Hessian of second derivatives is negative semi-definite everywhere.

4. Results

Proposition 1 (a) Given the optimal plan for the agent is to begin life working, retire at an age R, and not to work thereafter. (b) The retirement age and consumption stream that maximize expected life time utility are unique.

Proof in appendix 1.

Note that retirement emerges endogenously form our model. In our framework a lack of health effects on the disutility of labor and wages combined with exogenous wage growth, would tend to produce leisure when young and wages are low, and work when old, and wages are high; the health effects are essential for our model to produce retirement.

At the beginning of life the disutility of labor is low and benefits of working in terms of additional consumption spread over the life cycle are large. As life proceeds the rising disability eventually outweighs the benefits of working and retirement occurs. It is possible if wages are rising very rapidly that re-entry to the labor market occurs; we rule this out by limiting the rate of wage growth to be lower than the rate of increase in disability with age. Given proposition 1 we can rewrite the budget constraint as:

$$\int_0^T e^{-rt} s(t,z) c(t) dt \leq \int_0^R e^{-rt} s(t,z) w_1(t) w_2(h(t,z)) dt \qquad (18)$$

In what follows we think of the choice variables as the consumption level and the retirement age, chosen subject to equation (18). Consumption is steady over the life cycle. If wages are rising for young workers this can lead to dis-saving when young before saving in middle age for retirement when old as occurs in the examples shown in appendix 1.

Now consider what happens if life expectancy increases. Lee and Goldstein (2003) argue that a natural benchmark for responding to an increase in life expectancy is to increase the timing of all life course choices proportionately. Bloom, Canning, and Graham (2003) show that in the case of no uncertainty in the length of the lifespan, the optimal response to an increase in life span and health is to increase working life proportionately, keeping consumption unchanged, provided interest rates, time preference rates and exogenous wage growth are all zero. In our framework, however it is not clear that such a proportional response is even feasible. Consider an initial life expectancy z_0 and let the optimal retirement age be R_0^* and the optimal consumption level be c_0^*. Let the optimal retirement age at the longer life expectancy, z_1, be R_1^* and the optimal consumption level be c_1^*. We require that the old consumption stream is affordable at the new survival schedule provided retirement is postponed in line with life expectancy[6]. We call this the proportionality condition:

Proportionality Condition:

$$\int_0^T e^{-rt} s(t, z_1) c_0^* dt \leq \int_0^{R_0^* z_1 / z_0} e^{-rt} s(t, z_1) w_1(t) w_2(h(t, z_1)) dt \quad for\ z_1 > z_0 \qquad (19)$$

This proportionality condition may not be satisfied if the extension in life expectancy is due to a rise in the survival schedule concentrated in the period immediately after retirement. However if the gain in lifespan comes from a more general reduction in mortality rates that generate survival increases during the working life (which adds to labor income) or near the end of life (where consumption is heavily discounted), the additional income generated by

[6] Note that by proposition 1 we can summarize labor supply by a retirement age. We integrate labor income over time up to retirement to get the value of lifetime earnings.

proportionately later retirement will be sufficient to keep consumption unchanged. In practice this condition is satisfied in data from the United States. In Appendix 2 we examine cohort survival schedules for the United States (the survival curve faced by the cohort) for different birth years. We show that moving from the survival schedule of those born in 1900 to those born in 1925, 1950, 1975, or 2000 with an increase in the retirement age proportional to the increase in life expectancy, would in each case allow consumption to increase.[7] This means that keeping the working life proportional to life expectancy with consumption unchanged has indeed been feasible.

We begin by giving results for the relative compression of morbidity as set out in equation (3). We will later extend this result to the case where compression of morbidity is faster than proportional.

Proposition 2: If there is proportional compression of morbidity, and changes in the survival schedule satisfy the proportionality condition, the optimal response to an increase in life expectancy requires consumption to stay the same or increase.

See Appendix 3.

The intuition for this result is that it is possible for the agent to increase working life proportionately but keep consumption unchanged. In general the agent will do better than this. If both leisure and consumption are normal goods he will take some of the additional welfare in the form of consumption and some in the form of increased leisure.

[7] We use long run averages for the rate of wage growth and the real interest rate over the period.

Proposition 3: If there is proportional compression of morbidity, and changes in the survival schedule satisfy the proportionality condition, the optimal response to an increase in life expectancy requires the retirement age to rise less than proportionately with life expectancy.
See Appendix 4.

If $h(\rho t, \rho z) \gg h(t,z)$ rising life expectancy will increase wages and reduce the disutility of labor substantially in old age, potentially increasing the retirement age substantially. A more interesting case is near the border line where $h(\rho t, \rho z) = h(t,z)$ so that the "benchmark" response is a proportional rise in the retirement age keeping consumption unchanged. In this case the wealth effect of a rise in life expectancy will, as we have seen in proposition 2, increase consumption and as shown in proposition 3, decrease the proportion of life spent working.

As well as the possibility that health is rising faster than life expectancy, we have to guard against the possibility that wages are rising so rapidly over time that the substitution effect dominates the wealth effect and workers increase the proportion of life spent working to take advantage of the higher wages later in life.

We now turn to the issue of how retirement and consumption are affected by a change in the exogenous component of the wage schedule. We consider what happens when the levels of wages rises; we think of the wage schedule moving up proportionately at every point in time. We find that what happens depends crucially on the intertemporal elasticity of substitution, which in our model is the inverse of the coefficient of relative risk aversion because of the assumption of time-separable utility.

Proposition 4: When the level of wages rises the optimal response of the agent is to

(i) keep retirement unchanged and increase consumption proportionally with wages if the utility function has a local intertemporal elasticity of substitution of unity.
(ii) increase the age of retirement and increase consumption more than proportionally with wages if the utility function has a local intertemporal elasticity of substitution greater than unity.
(iii) decrease the age of retirement and increase consumption less than proportionally with wages if the utility function has a local intertemporal elasticity of substitution less than unity.
See Appendix 5.

It is well known that the impact of wages on consumption and the demand for leisure has both income and substitution effects. The pivotal case occurs when we have a relative risk aversion which is locally unity. We state this proposition to show that the result applies to our model. The long-term decline in the retirement age, and increasing in retirement savings, is consistent with an intertemporal elasticity of substitution less than unity (which in out model implies a coefficient of relative risk aversion that exceeds one).

We focus on the effects of changes in life expectancy and the level wages on retirement and saving decisions. Bloom, Canning and Moore (2004) also investigate the effects of small changes in the interest rate, rate of time preference, and the rate of wage growth, under highly restrictive functional forms for utility and mortality.

5. Implications for Social Security

Our results give the optimal retirement and savings behavior of fully rational agents under complete capital markets. In practice, imperfect capital markets (particularly annuity markets), lack of foresight about the need to save for retirement, and time inconsistency in preferences, will distort private decisions generating a role for social security systems (Hubbard and Judd 1987; Feldstein 1985; Laibson 1998; Laibson et al. 1998). In these circumstances our model will fail to predict behavior, but it can still be thought of as a benchmark for the design of social security systems whose objective is to mimic the optimal plan.

To take a simple example suppose the agent is completely myopic, maximizing utility at each point in time with no thought for the future. In this case we add a no borrowing constraint, but allow saving. Without this constraint the agent will borrow the maximum possible and consume it at time zero. The myopic agent will consume their entire wage income if they work, and will work at age t providing

$$u(w(t)) - v(h(t,z)) \geq u(0) \qquad (20)$$

That is, they work if the utility of the wage earned minus the disutility of working exceeds the utility of zero consumption. Such an agent will work until death if the utility of zero consumption is sufficiently small. Suppose now we have a compulsory defined contribution scheme that is designed to mimic the optimal policy. There will be forced savings while working leaving the agent enough to consume at the optimal level c^* as defined in the optimal policy above. At retirement the agent gets a annuity based on their contributions. Note that this has to be an annuity, a lump sum will be spent immediately leaving the agent destitute.

If the agent is allowed to choose their retirement age they will retire at age R when

$$u(c^*) - v(R,z) = u(a(R)) \qquad (21)$$

where $a(R)$ is the level annuity received if retiring at age R. They work up to the point where the utility of working and consuming c^* (what is left of income after forced savings) minus the disutility of working equals the utility from the retirement annuity without working. In the optimal policy derived above however, consumption is flat over the lifespan. This requires the agent chooses the retirement age so that $a(R^*) = c^*$. However given equation (21), this is impossible if working has any disutility, and the agent will always retire before the optimal retirement age. Even though the annuity benefit from early retirement is lower then the consumption when working, the agent will retire to avoid the disutility of working. With forced savings the myopic agent retires early to stop working and get access to these savings, and does not take into account that a longer working life will mean an increased annuity payout in the future. For full optimality we need both forced savings and a minimum retirement age set at the optimum point where the payout from the annuity equals the consumption level (the wage minus the forced saving) before retirement.

When life expectancy rises, existing social security arrangements often become financially unsustainable. A common argument is that to resolve this problem we need some combination of increases in contributions, reduction in benefits, and extension in the working life, dividing the "pain of adjustment" across all three margins. The problem with this "balanced" approach is that it increases leisure, since the retirement age does not go up as fast as life expectancy, while reducing consumption, both when working (to pay the higher contributions) and in retirement (due to lower benefits).

Our analysis suggests that an increase in life spans enlarges the budget set and the optimal response to this wealth effect is an increase in both leisure and consumption. To mimic this social security should lower contribution rates and increase benefit rates while raising the retirement age. Financial sustainability should be maintained entirely through a longer working life, though the effects of compound interest and rising wages over time mean that the working life extension can be less than proportional to the increase life expectancy.

Of course this assumes that the social security system is trying to mimic the individually optimal plan. The system may also have a goal of redistribution of income within a cohort or across cohorts (Kotlikoff, Spivak and Summers (1982)). With redistribution, the social planner may face a different budget constraint than the individual, and unless the transfers are lump sum, the implicit tax and subsidy rates will distort behavior. While defined benefit schemes often have a strong element of redistribution, defined contribution schemes are essentially forced saving that aim to overcome myopia on the part of the individual. Our analysis in this section applies to such defined contribution schemes.

In our analysis the optimal response to increasing life spans is mainly through increases in the retirement age. However, mandatory retirement at a fixed age may prevent individuals from responding optimally to longer life spans through longer working lives. While few social security systems have mandatory retirement, substantial evidence indicates that retirement in industrial countries clusters around specific ages that depend on retirement incentives inherent in the national social security system (Gruber and Wise 1998). If the institutional retirement age does not rise with life expectancy individuals may be forced to save more for their long retirement period. Bloom, Canning, Mansfield, and Moore (2007) examine empirically how changes in life expectancy affect savings behavior under different social security arrangements.

6. Conclusion

Our model demonstrates two major, long-term influences on the optimal age at retirement and on savings behavior. First, at higher levels of lifetime wages, the desire for increased leisure leads to early retirement funded through a higher savings rate. Second, longer life spans and healthier lives lead to a less than proportional increase in working life and lower savings rates.

We make a number of simplifying assumptions to derive our results. Our model assumes that the instantaneous utility function is additively separable in leisure and consumption, that the lifetime utility function is strongly intertemporally separable, and that the interest rate equals the rate of time preference. We also treat health as changing deterministically with age. A better model would have random health shocks, raising issues of precautionary saving for health care costs. Relaxing these assumptions provides promising directions for future work.

Appendix 1

Proposition 1: (a) Given our assumptions the optimal plan for the agent is to begin life working, retire at an age R, and not to work thereafter.

At point R where the agent is indifferent between working and retiring and the equality condition of equation (17) holds. Given equation (16) and the fact that $r = \delta$, optimal consumption is constant over the lifespan. Now consider the function

$$\psi(R) = u'(c)w_1(R)w_2(h(R,z)) - v(h(R,z)) \qquad (22)$$

Clearly for R to be the optimal retirement and satisfy equation (17) we require $\psi(R) = 0$. We wish to show that only one R can satisfy the condition that $\psi(R) = 0$ so the retirement age is unique and retirement is permanent. We do this by showing that $\psi'(R) < 0$ whenever $\psi(R) = 0$. Thus every point where $\psi(R) = 0$ is a transition from working to retirement and re-entry to the labor market once retired is impossible.

We proceed by contradiction. Assume $\psi'(R) \geq 0$. We have

$$\begin{aligned}\psi'(R) &= u'(c(R))\frac{dw_1(R)}{dR}w_2(h(R,z)) \\ &+ u'(c(R))w_1(R)\frac{dw_2(h(R,z))}{dh}\frac{dh(R,z)}{dR} - \frac{dv(h(R,z))}{dh}\frac{dh(R,z)}{dR} \geq 0\end{aligned} \qquad (23)$$

Hence at any R with $\psi(R) = 0$ we have

$$\begin{aligned}&\frac{d\log(w_1(R))}{dR} + \frac{d\log(w_2(h(R,z)))}{dh}\frac{dh(R,z)}{dR} \\ &\geq \frac{d\log(v(h(R,z)))}{dR}\end{aligned} \qquad (24)$$

Since $h(R,z)$ is decreasing in R while $w_2(h)$ is increasing in h, the second term on the left hand side of equation (24) is negative. Since the growth of the exogenous component of wages and the disutility of labor with respect to the retirement age is the same as with respect to time, equation (24) implies:

$$\frac{\dot{w}_1}{w_1} \geq \frac{\dot{v}}{v} \tag{25}$$

which contradicts the assumption shown in equation (11).

Hence $\psi'(R) < 0$ at every age R with $\psi(R) = 0$. It follows that ages that satisfy the first-order condition (where $\psi(R) = 0$) are isolated. Now take two adjacent points $R_1 < R_2$ that satisfy $\psi(R) = 0$. By the fact that the derivative of ψ is positive at both points we can find ε small such that $\psi(R_1 + \varepsilon) < 0$ and $\psi(R_2 - \varepsilon) > 0$. Hence by the intermediate value theorem there exists R_3 between R_1 and R_2 with $\psi(R_3) = 0$. This contradicts R_1 and R_2 being adjacent. It follows that there can be at most one age R satisfying $\psi(R) = 0$ and the worker works up to this age and does not work afterwards. We call R the retirement age.

Now consider the boundary conditions. These imply the worker does not work at T. Now suppose the worker never works. Then consumption is always zero and the other boundary condition implies that the workers prefers to work at time zero, a contradiction.

Therefore the worker starts life working, retirement occurs once, and is permanent.

(b) The retirement age and consumption stream that maximize expected life time utility are unique. Note that in the proof of part (a) we used the fact that in an optimal plan consumption is flat over time. We now, however, compare two optimal plans, one with a longer life expectancy and one with a shorter one. By part (a) each optimal plan has a unique retirement age. From the budget constraint it is clear that a later retirement age will generate higher lifetime consumption. Again we consider the function $\psi(R)$ over different possible retirement ages

$$\psi(R) = u'(c(R))w_1(R)w_2(h(R,z)) - v(h(R,z)) \tag{26}$$

We require $\psi(R) = 0$ at an optimal retirement age. Once again assume $\psi'(R) \geq 0$ we have

$$\psi'(R) = u''(c(R))\frac{dc(R)}{dR}w_1(R)w_2(h(R,z)) + u'(c(R))\frac{dw_1(R)}{dR}w_2(h(R,z))$$
$$+u'(c(R))w_1(R)\frac{dw_2(h(R,z))}{dh}\frac{dh(R,z)}{dR} - \frac{dv(h(R,z))}{dh}\frac{dh(R,z)}{dR} \geq 0 \qquad (27)$$

Now however, we have $\frac{dc(R)}{dR}$ >0 since we are comparing retirement over different optimal paths rather multiple retirement ages within a single optimal path. Now using the fact that $\psi(R) = 0$ at each retirement age we have

$$\left[\frac{-1}{\theta(c(R))}\right]\frac{d\log(c(R))}{dR} + \frac{d\log(w_2(h(R,z)))}{dh}\frac{dh(R,z)}{dR} + \frac{\dot{w}_1}{w_1} \geq \frac{\dot{v}}{v} \qquad (28)$$

where $\theta(c(R))$ is the local intertemporal elasticity of substitution at retirement, which is positive since the utility function is concave. Since $c(R)$ is increasing and $w_2(R,z)$ is decreasing in R, the first two terms in the left hand side of equation (28) are negative. Hence we again have that equation (25) must hold, which contradicts our assumptions. It follows that in this case we also have $\psi'(R) < 0$ at each optimal retirement age. By a similar argument to that it part (a) we have that $\psi(R) = 0$ only once and there is only one optimal plan for the agent.

Appendix 2

Proportionality Condition:

To investigate this we look at the empirical survival schedules for the United States. Let us assume that there is no health effect on wages; the health effect will increase wage income when life expectancy rises making the proportionality condition easier to satisfy. We assume real wages grow at the rate a constant rate σ. Then the budget constraint gives us the initial ratio of consumption to wages at the original survival schedule

$$\frac{c_0^*}{w_0} = \frac{\int_0^{R_0^*} e^{(\sigma - r)t} s(t, z_0)\, dt}{\int_0^{\infty} e^{-rt} s(t, z_0)\, dt} \tag{29}$$

This implies that the proportionality condition on the initial consumption-wage ratios is:

$$\frac{\int_0^{R_0^* z_1 / z_0} e^{(\sigma - r)t} s(t, z_1)\, dt}{\int_0^{\infty} e^{-rt} s(t, z_1)\, dt} \geq \frac{\int_0^{R_0^*} e^{(\sigma - r)t} s(t, z_0)\, dt}{\int_0^{\infty} e^{-rt} s(t, z_0)\, dt} \tag{30}$$

This can be easily checked given two survival schedules provided we have data for the real rate of interest, the rate of real wage growth, and the initial retirement age. In its forecasts for social security, the Congressional Budget Office (2004) uses historical averages for the real interest rate of 3.30% per annum and of the rate of real wage growth of 1.27% per annum; we use the same figures. Gendell (2001) shows that between 1950 and 2000 the mean retirement age for men fell from just under 69 years to just under 63 years of age. Costa (1998a) shows that labor force participation rates for men over 65 before 1900 were around 80% but had fallen to around 50% by 1950, suggesting a high mean age of retirement before 1900. We take a range of possible figures for the retirement age in 1900: 80, 70, and 60 to allow for variations across individuals.

We use data on survival rates for cohorts born in 1900, 1925, 1950, 1975, and 2000 based on mortality data and projections from the Office of the Actuary of the Social Security Administration. These data are available from the Berkeley Mortality Database.[8] Figure 1 shows the survival curves for each of these cohorts. The life expectancy of men in these cohorts rises from 51.5 years in 1990 to 78.2 by the year 2000.[9]

For each fixed retirement age we ask if a proportional increase in the age of retirement (proportional to life expectancy) would allow the same consumption stream as before when given a later survival schedule. Table 1 shows the results of such a comparison exercise starting in 1900. We assume that adult life starts at age 16. The first row in the table gives the years we are using for comparison purposes. The second row shows life expectancy at age 16 in each year. The third row gives the ratio of life expectancy at 16 in each year relative to the figure for 1900. This ratio is how much the retirement age has to be increased if the ratio of working life to adult life is to be kept constant.

We now turn to our calculation of the initial consumption-wage ratios. The third column of the fourth row gives initial consumption to wage ratio (at age 16) for someone born in 1900 who starts working at 16 and retires at age 60. This is calculated using the right-hand side of equation (29) above based on the survival schedule for someone born in 1900, with a real interest rate of 3.3% and real wage growth of 1.27% per year. Note that the initial consumption-wage ratio is 1.14 so that the worker (initially) consumes 14% more than his wage income. This is a

[8] available online at http://www.demog.berkeley.edu/~bmd/

[9] These cohort life expectancies are larger than the corresponding period life expectancies since the cohort figures take into account falling mortality rates over time as the cohort ages, while the more common period life expectancy is based on current cross sectional mortality rates.

common feature of optimal consumption when real wages are rising. The higher expected wages later in working life allow high initial consumption. The young worker borrows initially to allow high consumption but consumes less than his wage income as an older worker to repay this borrowing and save for retirement.

The next column of row 4 shows the initial consumption wage ratio the worker could afford if he had the survival schedule of someone born in 1925, but also increased his retirement age by 11% to compensate for the longer adult life expectancy. With this longer working life and total lifespan, the initial consumption-wage can rise to 1.19 indicating that the proportionality condition is satisfied. We repeat the analysis comparing the cohort born in 1900 with the cohorts born in 1950, 1975 and 2000 and in each case find the proportionality condition is satisfied. We also find that in pair wise comparisons (not reported) of earlier versus later cohorts based on the survival schedules for cohorts born in 1925, 1950, 1975 and 2000 the proportionality condition still holds.

Appendix 3

Proposition 2: If changes in the survival schedule satisfy the proportionality condition, the optimal response to an increase in life expectancy requires consumption to stay the same or increase.

Proof . Let R_o^*, c_0^* be the optimal retirement-consumption plan at life expectancy z_0^* and let R_1^*, c_1^* be the optimal retirement-consumption plan at life expectancy z_1^*. We derive a proof by contradiction.

Let us assume $c_1^* < c_0^*$. Let $z_1 = \theta z_0$ where $\theta > 1$.We must have $R_1^* < R_0^* z_1 / z_0 = \rho R_0^*$, giving more leisure, else the optimal plan cannot be as good as the proportional plan (c_0^*, pR_0^*) which is feasible. At the optimal retirement ages we have from the first-order condition (17) we have

$$u'(c_0^*) w_1(R_0^*) = \frac{v(h(R_0^*, z_0))}{w_2(h(R_0^*, z_0))}$$

$$u'(c_1^*) w_1(R_1^*) = \frac{v(h(R_1^*, z_1))}{w_2(h(R_1^*, z_1))}$$

(a) We first take the case where $R_1^* \geq R_0^*$. Since $R_1^* < \rho R_0^*$, v is strictly increasing and w_2 is strictly decreasing it follows that

$$\frac{v(h(R_1^*, z_1))}{w_2(h(R_1^*, z_1))} < \frac{v(h(\rho R_0^*, \rho z_0))}{w_2(h(\rho R_0^*, \rho z_0))} < \frac{v(h(R_0^*, z_0))}{w_2(h(R_0^*, z_0))}$$

Hence $u'(c_1^*) w_1(R_1^*) < u'(c_0^*) w_1(R_0^*)$. Further since $R_1^* \geq R_0^*$ we have $w_1(R_1^*) \geq w_1(R_0^*)$ since w_1 is increasing, hence $u'(c_1^*) < u'(c_0^*)$ and so $c_1^* > c_0^*$, a contradiction.

(b) Now we examine the case $R_1^* < R_0^*$ Again assume $c_1^* < c_0^*$.

$$u'(c_1^*)w_1(R_0^*) > u'(c_0^*)w_1(R_0^*) = \frac{v(h(R_0^*, z_0))}{w_2(h(R_0^*, z_0))} > \frac{v(h(R_0^*, z_1))}{w_2(h(R_0^*, z_1)}$$

By our optimality conditions (17) this implies that the agent with life expectancy z_1 works at time R_0^*. This is a contradiction with $R_1^* < R_0^*$ and proposition 1 which ensures non-re-entry into the labor market after retirement age is unique. Hence if $R_1^* < R_0^*$. we must have $c_1^* \geq c_0^*$.

Appendix 4

Proposition 3: If changes in the survival schedule satisfy the proportionality condition, $h(\rho t, \rho z) = h(t, z)$, and there is no exogenous wage growth, the optimal response to an increase in life expectancy requires the retirement age to rise proportionately, or less than proportionately, with life expectancy.

Proof: We prove by contradiction. Let R_o^*, c_0^* be the optimal retirement-consumption plan at life expectancy z_0^* and let R_1^*, c_1^* be the optimal retirement-consumption plan at life expectancy z_1^*. Set $\rho = z_1 / z_0$. Assume that $R_1^* > \rho R_0$. Hence $h(R_1, z_1) < h(\rho R_0, \rho z_0) = h(R_0, z_0)$.

At the optimal retirement ages we have from the first-order condition (17)

$$u'(c_0^*) w_1(R_0^*) = \frac{v(h(R_0^*, z_0))}{w_2(h(R_0^*, z_0))}$$

$$u'(c_1^*) w_1(R_1^*) = \frac{v(h(R_1^*, z_1))}{w_2(h(R_1^*, z_1))}$$

Since v is strictly increasing and w_2 is strictly decreasing

$$\frac{v(h(R_1^*, z_1))}{w_2(h(R_1^*, z_1))} > \frac{v(h(R_0^*, z_0))}{w_2(h(R_0^*, z_0))}$$

Since there is no exogenous wage growth (that is $w_1(R_0) = w_1(R_1)$) this implies $u'(c_1^*) > u'(c_0^*)$ which in turn implies $c_1 < c_0$. But this contradicts proposition 2. Hence $R_1^* \leq \rho R_0$.

Appendix 5

Proposition 4: When the wage schedule rises the optimal response of the agent is to

(i) keep retirement unchanged and increase consumption proportionally with wages if the utility function has a local intertemporal elasticity of substitution of unity.

(ii) increase the age of retirement and increase consumption more than proportionally with wages if the utility function has a local intertemporal elasticity of substitution greater than unity.

(iii) decrease the age of retirement and increase consumption less than proportionally with wages if the utility function has a local intertemporal elasticity of substitution less than unity.

Proof : Consider an increase in the whole time path of the wage schedule w_1 by a factor τ. We can then write the optimal retirement age and consumption level as function of the parameter τ, $R(\tau)$ and $c(\tau)$. The first order condition (17), which holds for all τ, is

$$u'(c(\tau))\tau w_1(R(\tau)) = \pi(R(\tau))$$

where

$$\pi(R(\tau)) = \frac{v(h(R(\tau), z_0)}{w_2(h(R(\tau), z_0)}$$

Clearly , the function π is increasing in R. Differentiating both sides of the first order condition we can derive

$$\frac{\tau R'(\tau)}{R(\tau)} \frac{R(\tau)\pi'(R(\tau))}{\pi(R(\tau))} - \frac{c(\tau)u''(c(\tau))}{u'(c(\tau))} \frac{\tau c'(\tau)}{c(\tau)} = 1$$

Or

$$\varepsilon_{R\tau}\varepsilon_{\pi R} + \frac{1}{\theta}\varepsilon_{c\tau} = 1 \qquad (31)$$

Where $\varepsilon_{R\tau}$ is the elasticity of the retirement age with respect to τ , $\varepsilon_{c\tau}$ is the elasticity of consumption with respect to τ , θ is the intertemporal elasticity of substitution, and $\varepsilon_{\pi R}$ is the elasticity of π with respect to R , which we know to be positive. All these elascticities are local though we suppress their dependence on consumption and age for convenience.

From the budget constraint given by equation (9) we have that the effect of increasing wages by the proportion τ is

$$c(\tau)\int_0^T e^{-rt}s(t,z)dt \leq \tau \int_0^{R(\tau)} e^{-rt}s(t,z)w_1(t)w_2(h(t,z))dt$$

It follows that changes to $c(\tau)/\tau$ and $R(\tau)$ have the same sign. Further, from this equation we can derive:

$$\begin{aligned} \varepsilon_{c\tau} = 1 &\Leftrightarrow \varepsilon_{R\tau} = 0 \\ \varepsilon_{c\tau} > 1 &\Leftrightarrow \varepsilon_{R\tau} > 0 \\ \varepsilon_{c\tau} < 1 &\Leftrightarrow \varepsilon_{R\tau} < 0 \end{aligned}$$

The proposition follows from these relationships and equation (31) .

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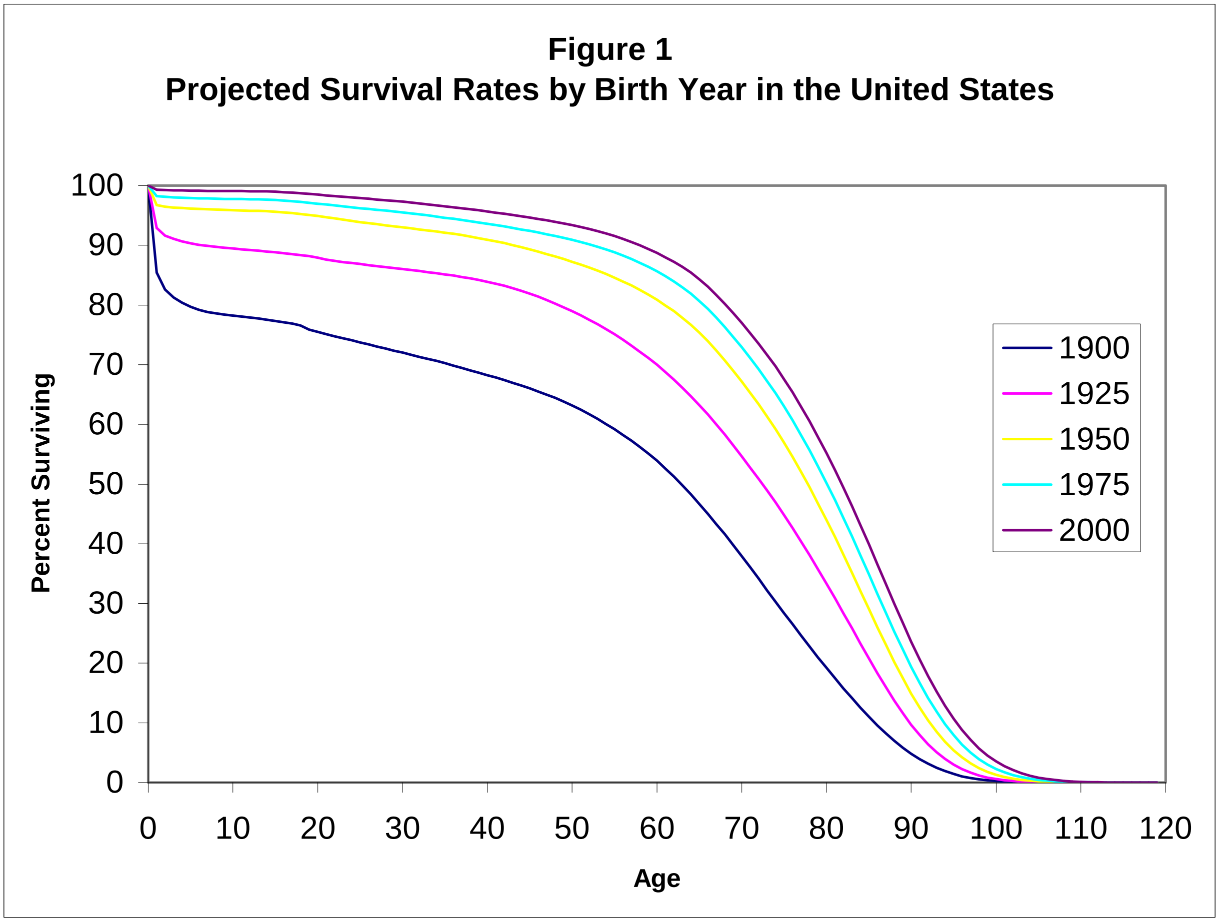

Source: Berkeley Mortality Database

Table 1

Testing the Proportionality Condition Relative to the 1900 Survival Schedule

	Retirement Age in 1900	Year of Survival Schedule				
Year		1900	1925	1950	1975	2000
Life Expectancy at Age 16		50.7	56.4	59.6	61.8	63.5
Life Expectancy Relative to 1900		1.00	1.11	1.18	1.22	1.25
Initial Consumption-Wage Ratio with retirement age proportional to life expectancy	60	1.14	1.19	1.22	1.25	1.26
	70	1.23	1.27	1.30	1.31	1.32
	80	1.28	1.31	1.32	1.33	1.34

For each retirement age, the proportionality condition is satisfied if the initial consumption wage ratio under the new survival schedule, allowing for an increase in the retirement age proportional to the increase in life expectancy, is no smaller than that with the 1900 survival schedule.

Source: Berkeley Mortality Database and the authors' calculations.