Absolute Abundance and Relative Scarcity: Announced Policy, Resource Extraction, and Carbon Emissions

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Abstract

We study the effectiveness of climate change policy in a model with multiple non-renewable resources that differ in their carbon content. We find that, when allowing some time between announcement and implementation of a cap on carbon dioxide emissions, emissions from non-renewable energy sources increase at the time of announcement. There are two channels behind this effect. First, since a binding constraint on emissions restricts energy use during some period of time, more must be extracted during other periods. Second, since low carbon energy sources are relatively more valuable when the policy is implemented, it is optimal to conserve them ahead of enforcement. This might induce a switch to high-carbon resources before the policy is implemented.

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Keywords: Climate Policy, Non-renewable Resources, Announcement Effects, Scarcity, Order of Extraction.

1 Introduction

It is common practice among policy makers to give firms and consumers time to adjust to proposed changes in environmental policies before they are actually enforced. By announcing policy measures well in advance, firms are given the opportunity to rethink and reorganize their production process – e.g. installing new production and abatement equipment – ahead of enforcement; consumers, likewise, may take future policies into consideration when deciding on the purchase of durable goods, for example. Thus, it is generally believed that announcing policy changes may reduce compliance costs.

It is likely, however, that the effects of policy announcements may be quite different in the context of energy and climate change policy. By increasing the price of (inputs based on) fossil fuels, climate policy should induce firms and consumers to substitute away from fuels and intermediate

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inputs with high carbon content. However, when faced with a lower demand for their products, fuels suppliers have an incentive to reduce their price. In particular, when the fuels are produced from non-renewable resources, the owners of carbon-containing resources have an incentive to lower their prices *in anticipation of future CO_2 reduction measures*, to prevent their resources from remaining unexploited. As anticipated policy leaves the agents free to emit in the period between announcement and implementation, it begs the question how carbon dioxide emissions in this period respond to the announcement of future climate policy.

Such concerns, however, have not entered the policy makers agendas, and there has been no attempt to implement climate change policy in an expedite way. The first commitment period of the Kyoto Protocol started on January 1, 2008, when agents had been aware for over ten years that a policy on greenhouse gas emissions was likely to come into force.

In this paper we study how emissions are affected by such an announcement, in the phase between the announcement itself and the policy's implementation. We use a model *à la* Hotelling (1931) in which utility is derived from consuming a final good (say energy), which is produced using two non-renewable resources differing in their carbon content (say coal and natural gas). We abstract from physical capital and focus on the optimal extraction path of the resources and the associated optimal emission path, following the announcement.

We show that announcement of climate policy induces two forces that might lead to an instantaneous *increase* in emissions. First there is an *abundance effect* of announcement on emissions, which stems from the fact that when during some period of time less of a non-renewable resource can be extracted, more of it must be extracted during other periods. Along the optimal path of energy consumption, some of this 'extra' energy will be consumed between the instants of the policy's announcement and implementation. The associated increase in energy consumption in this interim phase will then induce an increase in carbon dioxide emissions at the instant of announcement, if the carbon content of energy use does not fall compared to the laissez-faire economy. This abundance effect occurs independent of the number of resources in the economy.

In the case of multiple resources that differ in their carbon content, there may be an additional *ordering effect* from announcement, which increases the expected carbon content of energy use at announcement. When the resources are perfect substitutes, the composition i.e. the order of extraction is a matter of indifference in a laissez-faire economy. However, when the economy faces a binding constraint on emissions during some period of time, it is optimal to save some of the cleanest fuel for this phase, and mainly use the dirtier resource before the constraint becomes binding. As a consequence, the ordering of resource extraction changes compared to laissez-faire, and the expected carbon content of extraction increases in the period between announcement and implementation. This ordering effect never goes against the abundance effect, and might lead to a further increase in emissions at the instant of announcement. As the idea underlying the Kyoto Protocol is to stabilize the concentration of carbon dioxide in the atmosphere (Article 2 of the UNFCCC), the announcement effects of climate policy go directly *against* the spirit of the policy.

Surprisingly, only few papers study the effects of announced climate policy, and none has studied the effect of announcement on emissions. Kennedy (2002) and Parry and Toman (2002) focus on domestic climate policies in the period between announcement and implementation of international climate policy, and argue that policies aimed at emission reductions in this period may be costly and inefficient. Kennedy (2002) shows that, given a future ceiling on emissions, additional policy aimed at emission reductions before the future policy becomes enforced leads to too low investment in research and development and too much early capital investment, as the latter lead to immediate emission reductions whereas the former only lead to future emission reductions. Parry and Toman (2002) show that emission reductions before the commitment phase are efficient when banking of credits is allowed. This was not the case for the Kyoto Protocol. In this paper we do not look at additional emission reduction policies and only focus on the effects of announcement of the climate policy on emissions. Smulders and Van der Werf (2008) study the effect of climate policy on relative resource extraction when resources are imperfect substitutes and differ in carbon content, but the authors do not study the policy's effect on the level of energy use and the level of emissions.

The second key element of this paper, the optimal ordering of resource extraction, has been studied more intensively in the literature, starting with the seminal paper of Herfindahl (1967). In that paper it is shown, in a partial equilibrium context, that when resources only differ in their extraction cost, it is optimal to extract the lowest-cost resource first. Kemp and Long (1980) study under which conditions this result holds in a general equilibrium model. Lewis (1982) extends this to the case in which the resource can be converted in to productive capital, whereas Chakravorty and Krulce (1994) study heterogenous demand. Amigues, Favard, Gaudet, and Moreaux (1998), in turn, look at the case of a capacity constraint on the backstop technology.

In the context of climate policy, Chakravorty, Magné, and Moreaux (2006) discuss a ceiling on the stock of emissions, and show that in the case of one polluting resource and a clean backstop technology, the two inputs might be used jointly during the constrained phase, even though they are perfect substitutes. This analysis is extended to the case of two perfectly substitutable resources that differ in carbon content, in Chakravorty, Moreaux, and Tidball (2008). Neither paper studies the type of announcement effects that we focus on here as in their framework the (shadow) price of carbon is positive from the initial point in time. The latter paper shows that it may be optimal to initially use only the high-carbon input when it is abundant, and the cap is not initially binding in order to reach the ceiling as quickly as possible. In doing so, the social planner takes full advantage of the costless natural uptake of CO_2 by natural sinks like forests and the oceans. Our analysis shows that the ordering of resource extraction may be affected even in the absence of a positive carbon price, and that this result does not depend on the presence of natural uptake.

The remainder of this paper proceeds as follows. After we have introduced the model in section 2, we study the optimal path of energy consumption of an economy that never faces regulation (the laissez-faire economy). We then show in section 4 how announcement of climate policy leads to an abundance effect that induces an increase in energy use at the instant of announcement, which, when abstracting from ordering effects, leads to an increase in emissions. In section 5 we show under which conditions the two resources are abundant, and in section 6 we study the relative scarcity of the two resources. We then study how this affects the ordering of extraction after announcement. Section 8 shows how the abundance effect and the changes in the order of resource extraction affect optimal emission paths. We conclude in section 9.

2 The model

We study the optimal response of consumers, producers, and resource owners to an (announced) ceiling on the flow of carbon dioxide emissions. A high-carbon nonrenewable resource, H, and a low-carbon one, L, are perfect substitutes in the production of output R (electricity, say). The use of one unit of $i \in \{H, L\}$, entails the emission of ε_i units of carbon dioxide, Z, with $\varepsilon_H > \varepsilon_L$. The economy faces a constraint on the flow of CO_2 emissions, $Z \leq \bar{Z}$, at some exogenously set and known point in the future, $T \geq 0$.

Consumers derive utility from energy consumed. They maximize utility $U(R)$, which is a $\mathscr{C}^2$ function such that $U' > 0$ and $U'' < 0$, and satisfies the Inada conditions. Producers and resource owners maximize profits, taking prices and policies as given. Since there are no market failures in our model (as we study cost-effective rather than optimal climate policy), the decentralized economy can be represented by the social planner's solution in which utility is maximized subject to the technology, resource and emission constraints. Hence, the model reads:

$$\max_{\{R_H(t),R_L(t)\}_0^\infty} \int_0^\infty U(R(t))\,\mathrm{e}^{-\rho t}\mathrm{d}t \tag{1.a}$$

$$\text{s.t.}\quad R(t) = R_H(t) + R_L(t); \tag{1.b}$$

$$\dot{S}_H(t) = -R_H(t),\ R_H(t) \geq 0,\ S_H(0) = S_{H0}; \tag{1.c}$$

$$\dot{S}_L(t) = -R_L(t),\ R_L(t) \geq 0,\ S_L(0) = S_{L0}; \tag{1.d}$$

$$Z(t) \equiv \varepsilon_H R_H(t) + \varepsilon_L R_L(t) \leq \bar{Z}\ \forall t \geq T. \tag{1.e}$$

$R_i(t)$ denotes extraction of nonrenewable i at time (t), and ρ is the rate of time preference. Equations (1.c) and (1.d) show that the stock S_i of each nonrenewable declines with extraction. The initial endowment of each resource, S_{i0}, $i \in \{H, L\}$, is given. Throughout the paper, both the stocks and the extraction flows of the resources are expressed in units of energy.

Climate policy is described in (1.e): emissions (Z) arise from resource use, but from time T on they are constrained not to exceed $\bar{Z}$. The fact that the policy is announced at the beginning of the planning horizon but only becomes effective with a lag constitutes the key ingredient of our model. This entails the division of the planning horizon in two phases: a first period when the constraint is not yet enforced (the *interim phase*), and a second period when the constraint is enforced and (at least initially) binding (the *enforcement phase*). The problem in (1.a)-(1.e) is therefore an infinite-horizon discounted two-stage optimal control problem, with a fixed switching time at $t = T$. In the first stage, the problem faced by the agent is to optimally extract resources in order to maximize discounted utility over the interim phase, while leaving an *optimal amount* of resources for the second stage. Given the resource stock at the time of enforcement, in the second stage the agent will once more maximize discounted utility over the remaining horizon.

The Lagrangians for the two stages of the problem are:

$$\mathscr{L}^1(\cdot) = U(R(t)) - \sum_{i\in\{H;L\}} \lambda_i^1(t)R_i(t) + \sum_{i\in\{H,L\}} \gamma_i^1(t)R_i(t); \tag{2}$$

$$\mathscr{L}^2(\cdot) = U(R(t)) - \sum_{i\in\{H;L\}} \lambda_i^2(t)R_i(t) + \sum_{i\in\{H,L\}} \gamma_i^2(t)R_i(t) + \tau(t)\left(\bar{Z} - Z(t)\right); \tag{3}$$

where superscript 1 indicates the Lagrangian for the period $t \in [0, T)$, while 2 indicates the Lagrangian for $t \geq T$. The λ_i's are the co-state variables associated to (1.c) and (1.d), the γ_i's the multipliers for the nonnegativity constraints on the extraction rates, and τ is the multiplier associated with the emission constraint.

As the decision regarding the stock to bequeath to the second stage is made optimally, it is easy to show[1] that in this case the standard necessary conditions,[2]

$$U'(R(t)) = \lambda_i^j(t) - \gamma_i^j(t) + \varepsilon_i \tau(t) \equiv p_i(t), \tag{4}$$

$$\dot{\lambda}_i^j(t) = \rho \lambda_i^j(t); \tag{5}$$

where $i \in \{H, L\}$ and $j \in \{1, 2\}$, need to be complemented by the following matching conditions for the co-state variables in the two stages:

$$\lambda_i^1(T^-) = \lambda_i^2(T^+). \tag{6}$$

To simplify notation, in what follows we drop the superscripts to the λ's.

[1] See Appendix A.

[2] In the interest of compactness, we have indicated the necessary conditions for the two stages as one. Note that $\tau(t) = 0$ for all $t < T$.

The complementary slackness conditions for the constraints are,

$$\tau(t) \geq 0,\ \bar{Z} - Z(t) \geq 0,\ \tau(t)\left[\bar{Z} - Z(t)\right] = 0,\ \forall t \geq T; \tag{7}$$

$$\gamma_i^j(t) \geq 0,\ R_i(t) \geq 0,\ \gamma_i^j(t) R_i(t) = 0; \tag{8}$$

and the transversality conditions are,

$$\lim_{t\to\infty} \lambda_i(t) S_i(t)\, \mathrm{e}^{-\rho t} = 0. \tag{9}$$

3 The laissez-faire economy

As long as the constraint never binds, environmental policy has no influence on the agent's choices. For this reason, we refer to this benchmark case as the *laissez-faire* economy.

Since the two resources are perfect substitutes as sources of utility, the two non-renewables are *de facto* identical as long as their carbon content is irrelevant, and can be treated as one. For an economy that is never constrained, we can then define the total stock of available resources (in energy units) at time t as $S(t) \equiv S_H(t) + S_L(t)$, and the initial total stock as $S(0) \equiv S_0$.

Along the optimal extraction path, marginal utility grows in parallel with the scarcity rent λ at rate ρ, as can be seen from (4) and (5). At each point in time total extraction equals energy demand, and is simply given by $R(t) = d(\lambda(t)) \equiv U'^{-1}(\lambda(t))$. Thus, extraction is continuous and declines along the optimal path.

From (5) and (9) follows that it is not optimal to leave any of the resource in the ground, hence the stock will be exhausted over the infinite time horizon. The initial scarcity rent $\lambda_0 \equiv \lambda(0)$ then solves $\int_0^\infty d\left(\lambda_0 e^{\rho t}\right) \mathrm{d}t = S_0$, so that λ_0 is a function of S_0, $\lambda_0 = \Lambda(S_0)$. Hence, the larger the initial (total) resource stock, the lower λ_0, and the higher initial extraction (see (4)). In the laissez-faire economy, optimal extraction profiles are purely determined by the initial scarcity of the resource, hence we denote the optimal aggregate unconstrained extraction path by $\widetilde{R}(t; S_0) = d\left(\Lambda(S_0)\mathrm{e}^{\rho t}\right)$. Since the two resources are identical as long as their carbon content does not matter, the exact composition of extraction is undetermined, and so is energy's carbon intensity.

4 Optimal energy demand and policy announcements: the abundance effect

Suppose the emission constraint (1.e) were introduced without announcement, that is let $T = 0$. Then, provided the constraint is binding for some period, less of the resources can be extracted during this phase, compared to the laissez-faire extraction path. As a consequence, more of the resource is available after the constraint becomes slack (at time T_H, say) than would otherwise have been the case. In order for both stocks to be exhausted over time, extraction during this period needs to be larger than under laissez-faire.

When the policy is announced ahead of enforcement (i.e. $T > 0$), instead, the agents realize that the amount of resource that cannot be extracted during the constrained period, can be extracted not only after the constraint ceases to be binding, but also before the policy becomes enforced, i.e. in the interim phase. The resource aggregate thus becomes *de facto* more abundant already during the interim phase. Consequently, it is optimal to increase the amount of energy consumed in the $(0, T)$ interval, compared to the laissez-faire situation. This is, in a nutshell, the essence of the *abundance effect*:

Proposition 1 (The Abundance Effect). *Suppose an emission constraint is announced at* $t = 0$ *and becomes binding at* $t = T > 0$*. Then, resource use jumps up at the instant of announcement.*

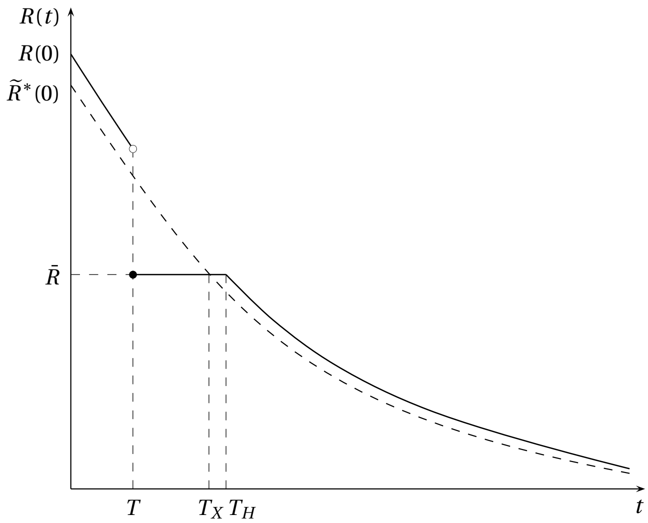

Figure 1: *Extraction paths for the laissez-faire economy – dashed line – and for the economy with an announced emissions constraint – solid line.*

Proof. By definition of a binding emissions constraint, $U(R(T)) < U(\widetilde{R}(T))$ implying $R(T) < \widetilde{R}(T)$. Thus, over some strictly positive time interval, $R(t) < \widetilde{R}(t)$. From (9), it then follows that during other periods $R(t) > \widetilde{R}(t)$.

From (5), marginal utility grows at rate ρ during all unconstrained phases along any optimal path. Thus if $U'(R(t))e^{-\rho t} \gtrless U'(\widetilde{R}(t))e^{-\rho t}$ for some $t \in \{(0,T),[T_H,\infty)\}$, the same holds for all t in the same interval.

Resource extraction has to be continuous at T_H or the Hamiltonian in (2) would be discontinuous, which cannot be optimal. Finally, $R(T) < \widetilde{R}(T)$; thus, it must be the case that $R(t) > \widetilde{R}(t), \forall t \in \{(0,T),[T_H,\infty)\}$, to satisfy the transversality conditions (9). □

From the necessary conditions, we know that the optimal path entails constant discounted marginal utility across all phases during which the economy is unconstrained – see (4) and (5). Hence the amount of the resource that comes available outside the constrained period due to the abundance effect must be extracted partly before and partly after the constrained phase.

An interesting aspect of the abundance effect is that at the instant of implementation ($t = T$), extraction (and hence utility) jumps down relative to the laissez-faire situation. This might be surprising since one might expect that risk-averse consumers would smooth their consumption over time, knowing in advance of the binding constraint. Consumption is smoothed wherever possible. Only at time T consumption discretely drops and marginal utility jumps up. However, this cannot be avoided since any reallocation of consumption towards the period just after T (with relatively high marginal utility) would result in violation of the emission constraint.

Figure 1 compares the extraction paths in the laissez-faire economy and in the economy with an announced constraint, for the special case in which all resources have the same emission intensity, or –equivalently– in which there is only one resource (note that the result in Proposition 1 holds for any number of resources, possibly with different carbon intensities). The level of extraction in the constrained economy is higher than under laissez-faire both before implementation at time T, and after $t = T_X$, the instant at which laissez-faire emissions drop below $\bar{Z}$.

From the proposition immediately follows that

Corollary 1. *Keeping emissions per unit of energy constant, the announcement of climate policy* *leads to an increase in emissions during the interim phase.*

Announcing climate policy leads to a perverse increase in emissions at the instant of announcement, if the emissions intensity of energy production does not fall. Note that the point made in the corollary can be strengthened: as Proposition 1 states that energy use *increases* ahead of implementation, so does the level of emissions, unless the carbon intensity of energy falls proportionally more than the increase in energy use.

In a setting with multiple resources, this emissions intensity is determined by the order, or composition, of extraction. In the following sections we investigate the case of two resources that differ in their carbon content to study whether our *cœteris paribus* assumption of constant emissions intensity holds in the case of multiple resources. We show that in addition to the abundance effect, there exists an *ordering* effect of the announcement on the carbon intensity of energy. As the optimal extraction paths are sensitive to initial conditions, however, it is first necessary to present a taxonomy of initial conditions in terms of the relative scarcity of the two resources.

5 Defining absolute abundance

The previous section described the abundance effect of an announced emissions constraint: provided that the constraint is binding for some time, energy consumption jumps up at the instant of announcement. For the constraint to be binding, however, it must be the case that initially so much of the resource is available that along a laissez-faire extraction path, more carbon dioxide would be emitted than allowed by the constraint. Hence, the abundance effect is directly linked to the initial *absolute abundance* of resources.

We now characterize resource endowments such that a given constraint will be binding for some period of time, and the abundance effect occurs. These are initial resource stocks such that laissez-faire extraction paths violate the emission constraint. By identifying all endowments such that a laissez-faire extraction path will leave the economy unconstrained, we obtain *a contrario* those that lead to an abundance effect.

To begin with, assume that the cap $\bar{Z}$ is immediately binding, i.e. let $T = 0$. In this case, it is possible to identify the largest stock of each type of resource $i \in \{H;L\}$ such that, if it were the only resource in the economy, its extraction leads to a path that is never constrained by a given ceiling on emissions $\bar{Z}$. Such a stock, which we denote by S_i^h, must be such that the associated optimal extraction path satisfies $\varepsilon_i \tilde{R}(0;S_i^h) = \bar{Z}$; i.e. the constraint is exactly binding at $t = 0$, and never again. Since these correspond to the largest possible stocks of each resource that may be extracted along an unconstrained (single-resource) path when the constraint is immediately enforced, we call these the *maximal Hotelling stocks*. Notice that $\tilde{R}(0;S_i^h) = \bar{Z}/\varepsilon_i \equiv \bar{R}_i$ is the maximum amount of i that can be extracted when the emission constraint is binding. For future use, it is useful to denote the price associated with $\bar{R}_i$ by $\bar{p}_i \equiv U'(\bar{R}_i)$. From the properties of U and the fact that $\varepsilon_H > \varepsilon_L$, it follows that $\bar{p}_H > \bar{p}_L$. Finally, since the emission coefficient determines the amount of resource that can be used at the ceiling, the cleaner the resource, the larger the maximal Hotelling stock, i.e. $S_L^h > S_H^h$.

These definitions are useful to identify initial endowments that lead to an unconstrained path when $T = 0$. We refer to Figure 2, which has stocks of the low-carbon resource on the abscissa, and stocks of the high-carbon on the ordinate, both expressed in units of energy rather than in physical units.[3] We begin by indicating the maximal Hotelling stocks in the figure. When stocks are at point $A = (0, S_H^h)$, extraction equals $\bar{R}_H$, the price is $\bar{p}_H$, and the path of extraction is unconstrained from the instant at which stocks are in point A onwards. Next, focus on the iso-energy line

[3]Given this convention, lines with negative unit slope are iso-energy lines.

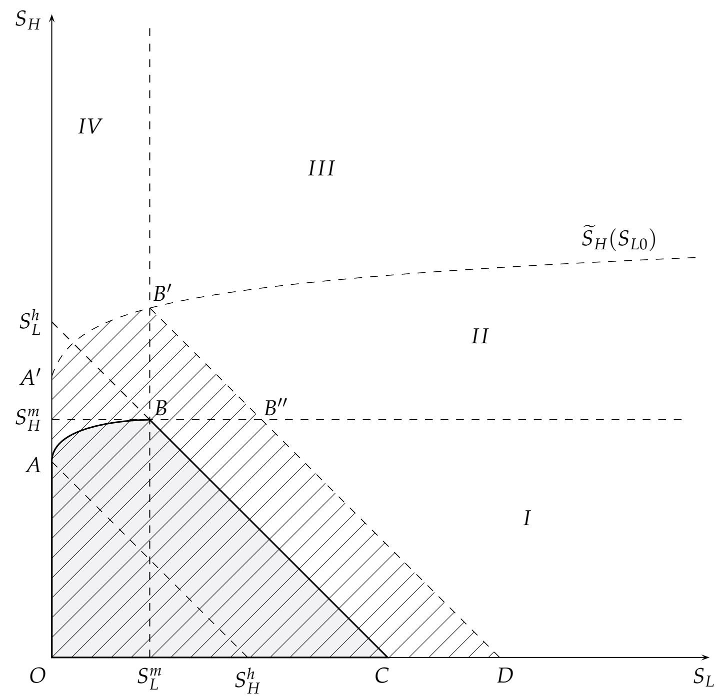

Figure 2: *A taxonomy of initial stocks*

through $C = (S_L^h, 0)$, which denotes all vectors such that $S_H(t) + S_L(t) = S_L^h$. The economy is unconstrained at point C by definition of maximal Hotelling stock. However, moving up along this line from point C, the carbon content of the vector of stocks increases. As a consequence, if initial demand (equal to $\bar{R}_L$) were satisfied using only the high-carbon source, emissions would exceed the cap. To avoid excessive emissions, enough of the low-carbon resource must be extracted at each point in time in combination with the high-carbon resource until the iso-energy line through A is reached and total stocks equal S_H^h. This requires a minimum amount of the low-carbon resource to be available initially. Along the iso-energy line through point C (i.e. when the total stock equals S_L^h), the minimum amount of low-carbon needed along an unconstrained path is S_L^m (which is defined in Appendix A), so that wherever initial stocks are on line segment BC, extraction is unconstrained. Starting from an iso-energy line with smaller total stock than S_L^h, unconstrained extraction is lower and correspondingly lower amounts of low-carbon resources are needed to make the unconstrained extraction path compatible with the emissions constrained. For any given total stock (i.e. along a particular iso-energy line), we can identify the minimum L-stock and the corresponding maximum H-stock. For $S_0 \in (S_H^h, S_L^h)$ this is represented by AB. In Appendix B we show that along the trajectory AB, emissions exactly equal $\bar{Z}$, and the price of energy increases from $\bar{p}_L$ to $\bar{p}_H$, as more and more R_H is used. Thus, when the vector of stocks is within the $OABC$ area, the emission constraint is never binding, and we can state the following:

Lemma 1. *When stocks are in area $OABC$ of Figure 2, the economy is unconstrained along an optimal extraction path.*

Proof. See Appendix B. □

When the policy is announced ahead of implementation, i.e. when $T > 0$, the possibility arises that during the interim phase some of the resources can be extracted without violating the constraint. Thus, in the presence of an announced constraint, the set of endowment vectors consistent with unconstrained paths is larger than the area identified above. The key issue is whether, for a given level of the constraint $\bar{Z}$, the vector of resource endowments lies within *$OABC$ at time T*.

Define by $\delta_H(T)$ the largest cumulative extraction of the dirty fuel that can be performed in the interim phase of an unconstrained path entailing the extraction of H only. Since the price of the dirty input at the beginning of the constrained period cannot be below $\bar{p}_H$ – the price that characterizes the maximal Hotelling path – the size of $\delta_H(T)$ only depends on the duration of the interim period, and is given by $\delta_H(T) = \int_0^T d(\bar{p}_H \mathrm{e}^{-\rho(T-t)})\mathrm{d}t$. In the same way, for $\delta_L(T)$ – the largest cumulative extraction of the clean fuel that can be carried out in the interim phase of an unconstrained path entailing the extraction of L only – we have $\delta_L(T) = \int_0^T d(\bar{p}_L \mathrm{e}^{-\rho(T-t)})\mathrm{d}t$. In terms of Figure 2, δ_H is represented by the vertical distance between A and A', whereas δ_L is the horizontal distance between C and D.

Let the vectors $\mathbf{S_0} = \{S_L(0), S_H(0)\}$ identify initial levels of the stock of resources. Recalling from section 3 that the optimal unconstrained extraction path associated with an initial total stock of resources equal to S_0 is $\widetilde{R}(t; S_0)$, we can define the set of all initial endowments leading to an unconstrained path as:

$$\mathbb{H} \equiv \{\mathbf{S_0} = \{S_L(0), S_H(0)\} : \sum_i R_i(t) = \widetilde{R}(t; S_0)\,\forall t \cap \sum_i \varepsilon_i R_i(t') \leq \bar{Z}\,\forall t' \geqslant T\}. \tag{10}$$

The following result relates the definition above to the graphical representation in Figure 2:

Lemma 2. *The set $\mathbb{H}$ is represented by the area $OA'B'D$ in Figure 2.*

Proof. See Appendix B. □

Thus, only initial endowments outside the set $\mathbb{H}$ can be considered *abundant*, in the sense that any path starting from such endowments will be constrained by climate policy. We can conclude that any initial endowment outside of set $\mathbb{H}$ will be subject to the abundance effect brought about by the announcement of the policy. Formally,

Proposition 2. *Any path beginning from a vector of initial stocks $\mathbf{S_0} \not\subset \mathbb{H}$ will be subject to the abundance effect.*

Proof. It immediately follows from the definition in (10) and Lemma 2. □

6 Relative scarcity and optimal extraction

Since the clean fuel allows more energy to be consumed during the constrained phase, agents prefer to conserve it for the enforcement phase. When L is available only in small amounts, it is attractive to use only the dirty fuel in the interim period, so that all of the clean one is still available when enforcement starts. However, when enough of the clean fuel is available from the start, it can be used already in the interim period, and still leave enough of it available for use at the time of enforcement.

If this intuition is correct, the introduction of an announced binding constraint has different effects on the relative scarcity of the two resources, depending on the initial endowment of each resource. As a first step, we show that the constraint never makes the high-carbon resource scarcer than the low-carbon one:

Lemma 3. *If the emissions constraint is binding for some period of time, then $\lambda_H(t) \leq \lambda_L(t)\ \forall t$ along any optimal extraction path.*

Proof. First, notice that if the cap is binding at time t, then $\tau(t) \geq 0$. Now suppose that $\lambda_H > \lambda_L$. From the necessary conditions (4) we must have: (i) if $\tau = 0$, only L is used; or (ii) if $\tau > 0$, then $p_H > p_L$, and again only L is used. Hence, H is never used, which, given (5), violates (9). Hence we must have $\lambda_H(t) \leq \lambda_L(t)\ \forall t$. □

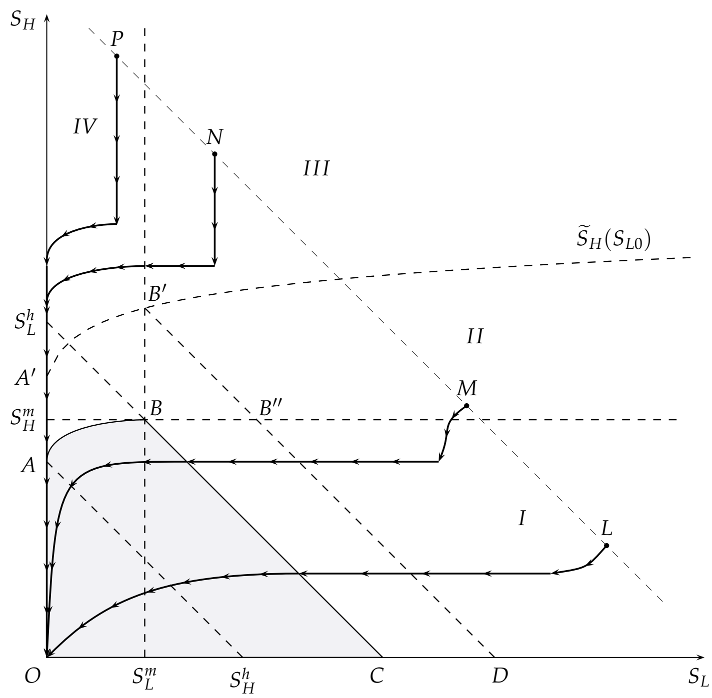

Figure 3: *Initial endowments and optimal extraction paths.*

Thus, the scarcity rent of the dirtier resource never exceeds the rent of the cleaner one. However, the optimal extraction path during the unconstrained phases will be very different depending on whether λ_H is equal to, or strictly smaller than λ_L: in the latter case, only the cheaper, dirtier fuel will be extracted during these phases. We now show how initial endowments determine the optimal extraction paths in the presence of announced climate policy. For ease of reference we draw these paths in Figure 3, for initial stocks characterized by the same total energy content. The proofs behind this Figure and the discussion below can be found in Appendix C.

6.1 Equal scarcity rents

Scarcity rents are equal when welfare cannot be improved by (marginally) changing initial endowments, replacing one unit of one resource by one unit of the other. This indifference situation arises, as we will now show, if the initial stock of L is large enough, while that of H is not too large. To fix ideas, let us consider the extreme situation where only the low-carbon resource is available. The optimal time path of extraction is as shown in Figure 1. The initial stock is too large to make an unconstrained path feasible, so extraction has to be capped from T to T_H. Now consider what happens when both resources are available. It is easy to see that the same path of total extraction (and thus the same welfare level) can be realized when the total energy stock is the same but (a sufficiently small) part of it is made up of the high-carbon resource. In particular, H could be extracted without problems once R falls below $\bar{R}_H$, as the cap does not bind any more then. Even when total energy use is between $\bar{R}_H$ and $\bar{R}_L$, however, H can be used to some extent, and still not violate the pollution constraint. More precisely, in the period after the constrained phase, i.e. after time T_H, an amount of H up to S_H^m can be used (see Lemma 2 and Figure 2).

This discussion allows us to define formally zone I as $\mathbb{S}_I \equiv \{\mathbf{S_0} : \mathbf{S_0} \not\subset \mathbb{H} \cap S_{H0} < S_H^m\}$, which elucidates that initial endowments in this zone are abundant in absolute terms such that the cap will bind at least over some period of time – the total stock is outside $\mathbb{H}$. However, the initial stock of the high-carbon fuel is small enough not to represent a constraint to extraction. Therefore, the cleaner resource L is *not scarce* relative to H, and hence $\lambda_H = \lambda_L$.

Along paths starting in zone I, from point L in Figure 3 say, the economy is constrained from time T until the instant when the path enters area $OABC$. During the constrained phase, only the low-carbon input is used in order to maximize energy supply (remember that $\bar{R}_L > \bar{R}_H$). Since the low-carbon input is very abundant, there is no need to save on it during the interim phase. As $\lambda_H = \lambda_L$, the composition of extraction during the interim phase is a matter of indifference, thus the path drawn in the picture is only one of the infinitely many possible ones.

Besides after T_H, extraction of H is of course also possible when the cap is not yet enforced, i.e. during the interim period. This allows for an expansion of the possible set of initial endowments such that scarcity rents are equal beyond zone I. Consider point M which has the same initial amount of aggregate energy as point L. From M it is thus possible to extract the same amount of energy that is optimal to extract when starting at point L, and end up in zone I before the cap is enforced. After that, aggregate extraction along both paths can clearly be the same. Since at most the amount S_H^m of the high-carbon resource can be extracted along an unconstrained path (see lemma 1), but more of H is available initially at points above zone I, it must be optimal to reduce this stock to at least S_H^m before the cap is enforced. How much of H can be extracted along the optimal path ahead of the constrained phase, however, depends on how much of L is initially available, as shown in Appendix C. For each level of S_{L0}, we can identify an upper bound on S_{H0}, which we denote by $\widetilde{S}_H$ in Figure 2, such that S_H^m can be reached before time T.

From this discussion, we can define zone II as $\mathbb{S}_{II} \equiv \{\mathbf{S_0} : \mathbf{S_0} \not\in \mathbb{H} \cap S_{L0} > S_L^m \cap S_{H0} \in (S_H^m, \widetilde{S}_H(S_{L0})]\}$. When initial stocks are in this zone, the dirty resource is relatively more abundant than in zone I and restricts the composition of extraction before the cap is enforced as described above. At the same time, the initial stock of H is small enough so that there is no need to use it exclusively before the cap is enforced: mixed use is still optimal, and $\lambda_L(t) = \lambda_H(t)$. Thus $\widetilde{S}_H(S_{L0})$ indicates the border between areas where the clean resource is abundant (zones I and II) and those where it is relatively scarce (zones III and IV).

To summarize, when $\mathbf{S_0} \in \mathbb{S}_{II}$, the optimal extraction paths are similar to zone I: initially the composition of extraction is indeterminate, provided that enough H is extracted to reach $\mathbb{S}_I$ at $t \leq T$; during the constrained phase only the low-carbon resource is used; finally, the composition of extraction is again indeterminate once $OABC$ is reached and the constraint ceases to be binding. In Figure 3, we draw one of the many possible paths starting from M.

6.2 Different scarcity rents

Initial endowments in zones III (defined as $\mathbb{S}_{III} \equiv \{\mathbf{S_0} : \mathbf{S_0} \not\in \mathbb{H} \cap S_{L0} > S_L^m \cap S_{H0} > \widetilde{S}_H(S_{L0})\}$) and IV (where $\mathbb{S}_{IV} \equiv \{\mathbf{S_0} : \mathbf{S_0} \not\in \mathbb{H} \cap S_{L0} < S_L^m\}$) are characterized by a relatively scarce clean resource. Along paths starting from these zones, scarcity rents are no longer equal, instead we have $\lambda_H < \lambda_L$. This implies, from the first-order conditions (4), that whenever the constraint is not binding (i.e. as long as $\tau = 0$) only the high-carbon content fuel will be used.

For simplicity, we begin with paths originating from zone IV (point P in Figure 3). In this case, the low-carbon input is scarce, in the sense that its endowment is too small to warrant its exclusive use during the constrained phase. Still, since it is the input that gives the highest amount of energy per unit of carbon, it is optimal to conserve as much as possible of it during the interim phase, hence in this phase only the high-carbon resource is used. At the beginning of the constrained phase, both fuels are used jointly until L is exhausted. For the rest of the constrained phase the dirty input is used at its maximum rate $\bar{R}_H$. Finally, when the trajectory of stocks enters $OABC$ at point A, the constraint ceases to bind, and extraction of H proceeds to exhaustion.

When $\mathbf{S_0} \in \mathbb{S}_{III}$, the extraction paths are similar to the ones discussed above: trajectories from initial stocks in zone III eventually enter zone IV and follow the path described above. Prior to entering zone IV, the optimal path is already constrained and only the clean fuel is extracted at the cap. In the interim phase it is optimal to save L and extract only H. Paths of this type are illustrated by the path starting from point N in Figure 3.

7 The ordering effect of an announced constraint

In section 4 we have shown that announcement of a constraint on the flow of emissions shifts extraction to the period before implementation if the total initial stock of resources is abundant enough to make the emissions constraint binding. In the previous section we have shown how the initial composition of the resource stock affects the composition and order of extraction. If initially the low-carbon resource is scarce relative to the total stock, announcement shifts extraction (partially, or – in case of unequal scarcity rents – completely) to the high-carbon resource in the interim period. In this case the announced constraint exerts what we will call an *ordering effect*.

7.1 The extreme ordering effect

Depending on initial endowments, the ordering effect occurs with different intensity. When initial stocks are in zone III or IV, total energy reserves are large and mainly consist of the high-carbon fuel. The large *total* stock provides the incentive to consume a lot during the interim phase. The *relative* scarcity of the low-carbon stock makes it optimal to maintain these high energy consumption levels without using the low-carbon fuel; L is indeed more valuable in the constrained period as it allows more energy consumption at the capped pollution level. As shown in section 6.2, the policy announcement makes the scarcity rent of low carbon resources exceed that of high carbon ones, hence it becomes optimal to extract only the high-carbon resource in the interim phase. We thus observe the ordering effect in its *extreme* form:

Proposition 3 (**The Extreme Ordering Effect**). *Suppose an emission constraint is announced at* $t = 0$ *and becomes binding at* $t = T > 0$. *Then, whenever* $\mathbf{S_0} \in \mathbb{S}_{III} \cup \mathbb{S}_{IV}$, *it is optimal to extract only the high-carbon resource* $\forall\, t \in [0, T)$.

Proof. As shown in Appendix C, paths initiated in zones III and IV are characterized by $\lambda_H < \lambda_L$. It follows from (4) and (8) that whenever $\tau = 0$, $R_H > 0$, and $R_L = 0$. □

The extreme ordering effect compounds the abundance effect in countering the intended outcome of the regulation in the interim phase: not only does resource use increase, it also becomes biased towards the most polluting fuel.

7.2 The weak ordering effect

Proposition 3 shows that for initial endowments in zone III or IV, the announcement induces an *extreme ordering effect,* as optimality requires that only the high-carbon input be extracted in the interim phase. On the other hand, section 6.1 shows that for initial endowments in zone I, the policy announcement does not make the clean resource relatively scarce (rents are equal for the two resources): enough of the clean resource is available (relative to the stock of the other resource) to allow exclusive use of it in the interim period and yet conserve enough for the constrained period. Therefore, just as in the laissez-faire economy, the ordering and composition of resource extraction are undetermined in the interim phase.

Between these two extremes, we find paths that start from zone II. Based on our discussion in section 6.1, we know that for such paths, although the scarcity rents are equalized, L is nevertheless scarce enough to dictate a precise requirement for extraction: optimality requires the path to enter

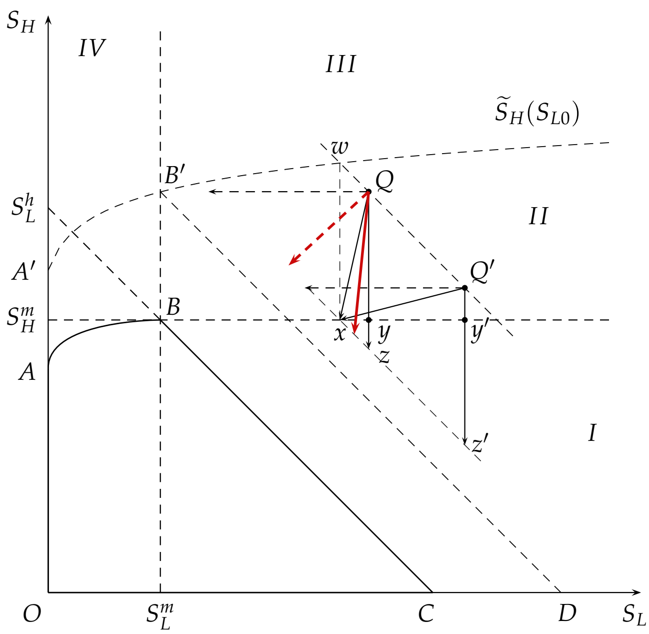

Figure 4: *Feasible paths during the interim phase for initial stocks in zone II.*

zone I by time $t = T$. This restriction imposes a lower bound on the amount of H that needs to be extracted in the interim phase, and *de facto* introduces an ordering effect, albeit in a weaker form than the one discussed in Proposition 3. Since L is not scarce enough to univocally determine the extraction path (as $\lambda_H = \lambda_L$, see section 6.1), however, such *weak ordering effect* can only be analyzed in terms of expected, rather than actual, carbon intensity. To see what the weak ordering effect entails, we now refer to Figure 4.

Consider any point Q in zone II. As discussed in section 3, under laissez-faire the exact composition of extraction at any point in time is indeterminate. Hence, the actual extraction path from Q for $t \in [0, T)$ can evolve in any direction within the quadrant to the South-West of the initial point. Assuming that any of these paths is equally likely, the 'expected' extraction path is given by the bisector of the SW quadrant (i.e. the dashed 45° vector to the SW from Q).

With announced climate policy, instead, the optimal extraction path originating from Q must enter zone I before the enforcement of the cap. This means that at least an amount of H equal to the vertical distance between Q and the horizontal S_H^m line in Figure 4 – the segment $\overline{Qy}$ – has to be extracted in the interim phase. Given that the total amount of energy that can be extracted during the interim phase is given by $\overline{wx}$ (the vertical distance between point w, where the iso-energy line through Q crosses the $\widetilde{S}_H$ locus, and the S_H^m line), at most $\overline{xy}$ of the clean input can be extracted before time T. Although it is impossible to be more precise as to which path will actually be followed, the composition of extraction will range from 100% H, along the vector $\overrightarrow{Qz}$, to the path with the lowest carbon content, along $\overrightarrow{Qx}$. The expected extraction path in this case it given by the bisector of the angle $\widehat{xQz}$, as depicted in the Figure. Notice that the range of possible optimal trajectories from Q after the announcement is reduced relative to laissez-faire; moreover, the trajectories that are not feasible are the ones with the lowest carbon content. Hence, we can conclude that the weak ordering effect induced by the announcement raises the carbon intensity of extraction during the interim phase, at least in expectations.

Proposition 4 (The Weak Ordering Effect). *Suppose an emission constraint is announced at $t = 0$ and becomes binding at $t = T > 0$. Then, whenever $\mathbf{S_0} \in \mathbb{S}_{II}$, the expected carbon intensity of extraction in the interim phase increases, relative to laissez-faire.*

Proof. In the text above. □

Finally, it is worth pointing out that, while the weak ordering effect operates everywhere in zone II, its impact is weaker, the lower the carbon content of initial stocks. Along any iso-energy line, initial endowments closer to zone I will be subject to a weaker ordering effect. This is illustrated in Figure 4 by point Q'. For initial stocks in Q', the range of possible paths is much larger than for point Q, since only a small amount of H needs to be extracted in the interim phase.

8 Announced policy and emissions

In section 4 we have shown that, irrespective of the number and carbon content of resources, energy use jumps up at the instant at which a future binding constraint on emissions is announced. This leads to an increase in emissions if the carbon content of energy extraction doesn't fall too much according to Corollary 1. We can now combine these results with the ones derived above on the ordering – and hence the carbon content – of extraction.

First, for initial stocks in zone I, we know that the abundance effect operates (Proposition 2), but there is no ordering effect at work. Although we cannot say anything specific in terms of actual extraction paths, there is no reason to expect that the announcement would affect the ordering of extraction compared to the laissez-faire economy. Thus, we expect that emissions increase at the instant of announcement.

When initial stocks are in zone II, the abundance effect is compounded by the weak ordering effect of Proposition 4. Given that the level of energy use increases following the policy announcement, and that the expected carbon intensity increases as well, we expect emissions to increase in the interim phase also in this case. Since the strength of the ordering effect increases with the proximity to the boundary between zone II and III ($\widetilde{S}(S_{L0})$ in Figure 4), our degree of confidence in this results also increases for endowments closer to zone III.

Finally, for initial stocks in zones III and IV, any ambiguity is resolved. For such initial endowments, Propositions 1 and 3 both imply that emissions increase in the interim phase. Not only does the level of extraction increase due to the abundance effect, but all of this extraction is made up of the most polluting resource due to the extreme ordering effect. Going strongly against the spirit of the regulation, emissions increase in the interim phase due to the policy announcement.

We summarize this discussion in the following proposition:

Proposition 5. *Suppose an emission constraint is announced at $t = 0$ and becomes binding at $t = T > 0$. Then, during the interim phase,*

1. $\forall\ \mathbf{S_0} \in \mathbb{S}_I$, *energy use increases due to the abundance effect, and emissions per unit of energy are expected not to be affected, leading to an* increase in expected emissions*;*
2. $\forall\ \mathbf{S_0} \in \mathbb{S}_{II}$, *energy use increases due to the abundance effect, and emissions per unit of energy are expected to increase, leading to an* increase in expected emissions*;*
3. $\forall\ \mathbf{S_0} \in \mathbb{S}_{III} \cup \mathbb{S}_{IV}$, *energy use increases due to the abundance effect, and emissions per unit of energy increase, leading to an* increase in emissions.

9 Discussion and conclusions

The purpose of climate policy under the UNFCCC is the "...stabilization of greenhouse gas concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system." (Article 2, UNFCCC) Projections of future emission paths for greenhouse gases all indicate that without emissions reduction policy, this goal will not be reached. An important means to reduce total greenhouse gas emissions is to reduce carbon dioxide emissions from the use of fossil fuels. Since fossil fuels are in essence nonrenewable resources, and since the owners of these resources would like to sell their entire resource stocks over time, this means that resource extraction has to be postponed relative to a laissez-faire economy.

In this paper we have shown that, although policy makers have good reasons to announce climate policies in advance – for example to reduce the burden of adjustment to firms – such an announcement has effects on CO_2 emissions that go *against* the spirit of the regulation.

When climate policy is announced, an *abundance effect* occurs, which increases the demand for energy at the time of announcement. We have shown that energy demand exceeds the laissez-faire's level until the constraint becomes binding: since a (binding) ceiling on carbon emissions implies that less energy is extracted over some time interval relative to the unconstrained case, more of it has to be extracted in other periods, including the period between the policy's announcement and its implementation (the interim phase). As a consequence, if energy's carbon intensity does not fall enough during this phase, carbon emissions will be higher than in the absence of regulation.

Whether the carbon content of extraction changes during the interim phase, depends on the relative extraction of high- and low-carbon resources. We have shown that whenever the low-carbon input is relatively scarce, only the high-carbon input is used during the interim phase. This is what we call the *extreme ordering effect*. When the low-carbon input is (relatively) less scarce, a *weak ordering effect* occurs, leading to an expected carbon content during the interim phase that is lower than its maximum level, but strictly higher than the expected carbon content of energy use in the laissez-faire economy. Finally, when the low-carbon input is not relatively scarce, no ordering effect materializes, and the expected emissions intensity is the same as under laissez-faire.

A quick look at the data suggests that our propositions have important implications for the design of future policies. If major emitters like the United States and China were to decide to restrict their carbon dioxide emissions, this might have significant effects on resource rents due to the scale of these countries. In these countries, as well as at the global level, the low-carbon input is indeed relatively scarce: the ratio of proved reserves of coal and natural gas is 22:1 in the US, 35:1 in China, and a bit less than 3:1 at a global level.[4] By the sheer abundance of such an high-carbon input as coal, the announcement of a future ceiling on emissions by these countries might then induce an increase in the relative scarcity rent of gas, and substitution towards coal before the emission ceiling is enforced. Indeed, several simulation studies have shown that with strict climate policy, the scarcity rent of coal drops sharply (see e.g. Chakravorty, Roumasset, and Tse, 1997).

As the abundance effect is always present when the total resource stock is abundant, and since it is compounded by a higher expected carbon intensity of energy use whenever the low-carbon input is relatively scarce, climate policy might lead to an adverse increase in emissions in the period between announcement and implementation, thus accelerating global warming. That is, announcement of a policy to stabilize concentrations of carbon dioxide in the atmosphere might have beneficial economic effects by reducing the burden of the policy, but might have environmental results that go directly against the policy's ultimate target. Our results therefore suggest that there is a trade-off with respect to climate policy: reducing the cost of compliance through pre-announcement of the policy, versus the environmental risk that this will increase carbon diox-

[4]We use data from the 2008 BP Statistical Review, and converted proved reserves into common units. Proved reserves are quantities that geological and engineering information indicates with reasonable certainty can be recovered in the future from known reservoirs under existing economic and operating conditions.

ide emissions. Recognizing the existence of this trade-off has consequences for countries that do not yet face binding carbon dioxide emission targets: the risk of inducing an increase in emissions should be taken into account.

If climate policy were to be set optimally, such trade-offs would indeed play a major role in policy design. While in our investigation we focus on the effect of the announcement of exogenously given policies on emissions, we can readily draw some implications for welfare as well. An announced policy that increases short-run emissions is likely to reduce welfare through its effect on damages from climate change. Indeed, Hoel (2006) shows that under reasonable conditions the emissions have to be postponed to reduce the cost of climate change. This suggests that the announcement effects we find are welfare-reducing and that optimal policies should therefore strive to minimize such effects. At the same time, pre-announcing policies has merits as noticed above, and can probably not be avoided given implementation lags. The issue is then to identify policies that do better in the presence of announcement effect. We expect that an announced tax policy, in the form of the announcement of a path for emission taxes starting at some future date, has the potential to reduce climate change costs. As long as taxes are (credibly) announced to decline sufficiently over time, the incentive to conserve resources for use when taxes are low outweighs the incentive to use them before implementation. Tax policies therefore seem to be better equipped to avoid undesirable announcement effects. The design of the optimal tax path has to take into account that resource owners tend to extract more in anticipation of future taxes. How exactly the announcement of tax policy affects the optimal carbon tax is left for future research.

There are further limitations of our analysis that open up avenues for future research. We omitted many relevant aspects of climate change and fossil fuel use such as extraction costs, limited substitution possibilities between different carbon inputs in specific sectors, and backstop technologies. However, as we show in the remainder of this section, our main results are likely to carry over to realistic cases that take into account these aspects.

Consider the benchmark case with constant marginal extraction costs (but perfect substitution), and assume that extraction costs are lower for the high-carbon input, say coal, than for the low-carbon input, say gas (the case of high-cost high-carbon resources does not seem empirically relevant). Then without climate policy, the ordering of extraction will be such that the high-carbon resource is extracted before extraction of the low-carbon resource starts, following the least-cost-first result of Herfindahl (1967). In response to climate change policy announcement, the abundance effect will still be present since the emission constraint makes the resources abundant independent of extraction costs. In this simple setting, however, the ordering effect cannot be in the direction of the high-carbon resource. Starting from a situation without policy where initially only the low-cost high-carbon resource is used, it is possible to show that the announcement of policy never induces a switch to the cleaner substitute in the interim phase, unless the high-carbon source is exhausted in the interim of the regulation. Thus, the average pollution intensity in the interim period cannot increase (as no ordering effect takes place). However a fall is also unlikely, as this would require an unrealistically small stock of the high-carbon resource. Indeed, the large stocks of coal available, and the short time-span of the interim phase (5 to 20 years) devoid this possibility of realism, and the extension of our model with constant extraction costs is unlikely to reverse our main conclusions.

The introduction of extraction costs has the advantage of removing the indeterminacy of the resource use mix in the unconstrained economy. However, it implies the unrealistic feature that resources are not used simultaneously. An interesting extension is therefore the introduction of limited possibilities for substitution. Then, coal and gas are both in use in the absence of climate change policy and the announcement of climate change policy may naturally trigger the ordering effect through substitution between coal and gas. From the calibration in Smulders and Van der Werf (2008) it is known that without extraction costs the substitution effect is likely to go in the direction of coal. This suggests that our main mechanisms (abundance and ordering effect) would also go through with imperfect substitution.

Including a backstop technology will not affect the main conclusions of the paper either. If the price of the backstop is sufficiently high to make the switch to the backstop occur after the emission ceiling ceases to be binding (which is for example assumed in Chakravorty, Moreaux, and Tidball, 2008), our results do not change at all. If the backstop price is lower, any changes to the extraction paths we describe in this paper, are to be found only during the constrained phase: if a switch to the backstop was made in the interim phase, then emissions would be zero from that instant onward, and the emission constraint would never be binding. Since our abundance and ordering effects occur in the period before the constraint becomes binding, use of the backstop in the constrained phase will not affect our results.

The discussion above shows that while many avenues remain open for possible extensions, the main insights of this paper are robust to many modification of the set-up. The most promising direction for future research in our view, however, is to reconsider the ranking of climate policy instruments when announcement effects are present.

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A Continuity of the scarcity rents

Let us proceed backwards. The second stage problem can be simply written as:

$$\max_{\{R_H(t),R_L(t)\}_T^\infty} \int_T^\infty U(R(t))\,\mathrm{e}^{-\rho t}\mathrm{d}t \tag{A.1}$$

$$\text{s.t. } R(t) = R_H(t) + R_L(t); \tag{A.2}$$

$$\dot{S}_j(t) = -R_j(t),\ R_j(t) \geq 0,\ S_j(T) \text{ given}; \tag{A.3}$$

$$Z(t) \equiv \varepsilon_H R_H(t) + \varepsilon_L R_L(t) \leq \bar{Z}. \tag{A.4}$$

Let $V_T(\mathbf{S}(T))$ be the *value-function* corresponding to the initial stock $\mathbf{S}(T) = \{S_L(T), S_H(T)\}$:

$$V_T(\mathbf{S}(T)) = \int_T^\infty U(\mathbf{R}^*)e^{-\rho(t-T)}\mathrm{d}t,$$

where $\mathbf{R}^*(t) = \{R_L^*(t), R_H^*(t)\}$. We then have (see for example the derivation of eq. 4.80 in Leonard and Van Long, 1992):

$$\frac{\partial V_T(\cdot)}{\partial S_j(T)} = \lambda_j^2(T), \tag{A.5}$$

where λ_j^2 is the costate variable associated with the state S_j in the second stage problem.

We can now write the first stage problem explicitly including the value-function $V_T(\mathbf{S}(T))$ as a scrap-value function, i.e.

$$\max_{\{R_H(t),R_L(t)\}_0^T} \int_0^T U(R(t))\,\mathrm{e}^{-\rho t}\mathrm{d}t + r^{-\rho T}V_T(\mathbf{S}(T)) \tag{A.6}$$

$$\text{s.t. } R(t) = R_H(t) + R_L(t); \tag{A.7}$$

$$\dot{S}_j(t) = -R_j(t),\ R_j(t) \geq 0,\ S_j(0) \text{ given}. \tag{A.8}$$

In this case, the transversality conditions read (Leonard and Van Long, 1992, Theorem 7.2.1):

$$\lambda_j^1(T) = \frac{\partial V_T(\cdot)}{\partial S_j(T)}. \tag{A.9}$$

Using (A.5) together with (A.9), and ruling out discontinuities in the stocks, yields (6). □

B Initial stocks and the abundance effect

B.1 Proof of Lemma 1

If, for given initial total resource stock S, we choose the Hotelling path for total extraction, and find it *feasible* to choose the composition of extraction so that $Z < \bar{Z}$, then it must also be *optimal* to follow such path, since the non-constrained path cannot be worse than any constrained one. Hence, we only need to prove that the unconstrained (Hotelling) solution is feasible in the constrained economy if we start in area $OABC$.

For the unconstrained problem (1.a)-(1.d), we denote the solution of total extraction at time t when the total stock at time 0 equals S_0 by $\tilde{R}(t, S_0)$. Since the problem is time autonomous, we can also write $\tilde{R}(\theta, \Sigma)$ for the solution of total extraction a period of length θ after $S = \sigma$ was reached, i.e. along an unconstrained path and for any t', we have $R(t' + \theta) = \tilde{R}(\theta, \Sigma)$ if $S(t') = \Sigma$. Finally, we define S_i^h, $i = L, H$, as the solution of $\tilde{R}(0, S_i^h) = \bar{Z}/\varepsilon_i$, i.e. at the time the total stock equals S_i^h, total unconstrained extraction equals $\bar{Z}/\varepsilon_i$.

1. If $S(t) < S_H^h$ (which is represented by a point to the left of the iso-energy line through A), then $\tilde{R}(t', S(t)) < \bar{Z}/\varepsilon_H$ and $Z(t') < \bar{Z}$ for all $t' \geq t$ along the Hotelling path. Hence the Hotelling path is feasible the in constrained economy.

2. If $S(t) > S_L^h$ (which is represented by a point to the right of the iso-energy line through C), then $\tilde{R}(\theta, S(t)) > \bar{Z}/\varepsilon_L$ so $Z(t+\theta) > \bar{Z}$ for $\theta > 0$ sufficiently small along the Hotelling path. Hence the Hotelling path is not feasible in the constrained economy.

3. If $S_L^h \geq S(t) \geq S_H^h$ (which is represented by any point between or on the iso-energy lines through A and C), then $\bar{Z}/\varepsilon_H \leq \tilde{R}(\theta, S(t)) \leq \bar{Z}/\varepsilon_L$ for $\theta > 0$ sufficiently small. For any $R \leq \bar{Z}/\varepsilon_L$, we have $Z = \varepsilon_L R_L + \varepsilon_H (R - R_L) \leq \bar{Z}$ if $R_L \geq \max\{0, (\varepsilon_H R - \bar{Z})/(\varepsilon_H - \varepsilon_L)\} \equiv R_L^z(R)$.

   Hence, $R_L^z(R)$ is the minimum amount of L needed to ensure $Z \leq \bar{Z}$ when total extraction equals R, and puts a lower bound on the extraction of L in the constrained economy. Since $R_L \leq R$, we note that $R_L^z(R)$ is defined only for $R \leq \bar{Z}/\varepsilon_L$. Hence, by construction, to make an unconstrained extraction path, with $S(t) \leq S_L^h$, compatible with $Z(t) \leq \bar{Z}$, we need at least $R_L^z(\tilde{R}(0, S(t)))$ of L at any time t. Therefore the minimum amount of L needed over time to make an unconstrained path that starts with $S(t) \leq S_L^h$ feasible in the constrained economy equals

$$\int_0^\infty R_L^z(\tilde{R}(\theta, S(t)))\mathrm{d}\theta.$$

   This integral equals zero if $S(t) < S_H^h$, since $R_L^z = 0$ for $R < \bar{Z}/\varepsilon_L = \tilde{R}(0, S_H^h)$, but this case was already discussed above; otherwise the integral equals:

$$\int_0^{t_H} R_L^z(\tilde{R}(\theta, S(t)))\mathrm{d}\theta \equiv S_L^z(S(t)),$$

   where t_H is the time it takes to reduce the stock from $S(t)$ to S_H^h along an unconstrained path, i.e. t_H solves $\tilde{R}(t_H, S(t)) = \bar{Z}/\varepsilon_H$.

   Thus $S_L^z(S)$ is the minimum amount of L needed to make the unconstrained extraction path feasible in the constrained economy, when starting from a total stock S. The corresponding maximum amount of H is then $S_H^z(S) = S - S_L^z(S)$. Now it follows immediately that the minimum L and maximum H that are needed to ensure a Hotelling path with total resources $S \in [S_H^h, S_L^h]$ can be realized in the constrained economy are given by the pairs $\{S_L^z(S), S_H^z(S)\}$. These pairs are represented by segment AB in the figure, of which the slope can be expressed as: $S_H^{z\prime}(S)/S_L^{z\prime}(S) = (\varepsilon_L \tilde{R}(0,S) - \bar{Z})/(\varepsilon_H \tilde{R}(0,S) - \bar{Z})$, so that the slopes of AB in points A and B are given by, respectively, infinity and 0. Hence in point B, $\tilde{R}(t, S_0) = \bar{R}_L$ and $p(t) = \bar{p}_L$; in point A, $\tilde{R}(t, S_0) = \bar{R}_H$ and $p(t) = \bar{p}_H$. Note that in the main text we define S_L^m as the minimum amount of L needed to make the Hotelling path that starts at $S = S_L^h$ feasible in the constrained economy, i.e. $S_L^m \equiv S_L^z(S_L^h)$.

B.2 Proof of Lemma 2

We are looking for the largest initial stocks, such that if an Hotelling path is followed from $t = 0$, the pollution constraint is not violated from T onwards. By lemma 1, this implies that stocks at $t = T$ must be on ABC. To allow unconstrained extraction between $t = 0$ and $t = T$, the initial total stock must equal $S(T) + \delta$ such that $S(T)$ is on ABC and δ is cumulative extraction following Hotelling between $t = 0$ and $t = T$. Let δ solve $\delta = \int_0^T \tilde{R}(t, S + \delta)\mathrm{d}t$ and call the solution $\delta(S, T)$. For any point (S_L, S_H) on ABC, $\delta(S_L + S_H, T)$ represents cumulative extraction along a Hotelling path over a period of length T that ends with total stock $S_L + S_H$. The maximal initial stocks are then represented by $A'B'D$ such that the vertical distance between any point (S_L, S_H) on ABC and $A'B'D$ equals $\delta(S_L + S_H, T)$. Note that the vertical distance between BC and $B'D$ equals $\delta(S_L^h, T) \equiv$

δ_L and the vertical distance between A and A' equals $\delta(S_H^h, T) \equiv \delta_H$. The vertical distance between AB and $A'B'$ increases the more we move to the right since points on AB further to the right are associated with higher S, and δ increases in S.

C Optimal extraction paths

Lemma C.1. *Along any optimal constrained path,* $\lambda_L(t) = \lambda_H(t)\ \forall t \in [0,\infty)$ *if and only if* $S_{L0} > S_L^m$, *and* $S_{H0} < \widetilde{S}_H(S_{L0}) \equiv S_H^m + \int_0^T d\left(\bar{p}_L e^{-\rho[T+(S_{L0}-S_L^m)/\bar{R}_L - t]}\right) \mathrm{d}t$.

Proof. Only if: Suppose $\lambda_L(t) = \lambda_H(t)$. Since $\mathbf{S_0} \notin \mathbb{H}$, there exists $\mathscr{T} = [T, T_H)$ such that $\tau(t) > 0$ $\forall t \in \mathscr{T}$, and $\tau(t) = 0$ elsewhere. Since $\lambda_L(t) = \lambda_H(t)$, then $R_L(t) = \bar{R}_L$ and $R_H(t) = 0\ \forall t \in \mathscr{T}$, see (4). From this and lemma 1, it follows that $S_L(T_H) \in [S_L^m, S_L^h]$, and $S_H(T_H) \in [0, S_H^m]$. Since $T_H > T$, $S_{L0} > S_L^m$.

As $S_L(T_H) \in [S_L^m, S_L^h]$, it follows that $\forall S_L(0) = S_{L0}$, at most $S_{L0} - S_L^m$ of L can be extracted during the constrained phase. Thus, this phase lasts at most $(S_{L0} - S_L^m)/\bar{R}_L$ periods. Since extraction must be continuous at T_H, it follows that $\lambda_H(T_H) = \lambda_L(T_H) = \bar{p}_L$. From (4) and (5), it follows that the maximum amount of H that can be extracted during the interim phase is:

$$\widetilde{\delta}_H(S_{L0}) = \int_0^T d\left(\bar{p}_L e^{-\rho[T+(S_{L0}-S_L^m)/\bar{R}_L - t]}\right) \mathrm{d}t,$$

where we have used $T_H = T + \frac{S_{L0}-S_L^m}{\bar{R}_L}$. Since $S_H(T_H) \in [0, S_H^m]$, and $R_H(t) = 0\ \forall t \in \mathscr{T}$, it follows that $S_{H0} < \widetilde{S}_H(S_{L0}) \equiv S_H^m + \widetilde{\delta}_H(S_{L0})$.

If: Suppose $\lambda_L > \lambda_H$. Assume that both S_{L0} and S_{H0} are positive.

We first note that then, whenever $\tau = 0$, we cannot have simultaneous use, see (4), and that we cannot have a switch from exclusive use of one resource to exclusive use of the other resource (since it would imply a jump in price and hence in extraction, so that the Hamiltonian becomes discontinuous). Hence, $R_L = 0$ whenever $\tau = 0$, and all L must be depleted when $t \in \mathscr{T}$ to satisfy the transversality condition (9).

Second, whenever $\tau > 0$, switching from exclusive use of H to mixed use, and from mixed use to exclusive use of L cannot be optimal:

- When $\tau > 0$, exclusive use of H use implies $R(t) = R_H(t) = \bar{R}_H$. A switch to mixed use implies either lower pollution (contradicts $\tau > 0$), or higher extraction (violating continuity) or both;
- When $\tau > 0$, exclusive use of L use implies $R(t) = R_L(t) = \bar{R}_L$. A switch to mixed use implies either higher pollution (violating the pollution constraint) or lower extraction (violating continuity) or both.

Third, we note that it cannot be optimal to use L immediately before the economy becomes unconstrained since any use of L at the cap implies $R(t) > \bar{R}_H$, while when $\tau(t) = 0$ only H is used, so that $R(t) \leq \bar{R}_H$ and $R(t)$ has to be continuous.

It follows that, when constrained, the economy uses both resources simultaneously until L is depleted, followed by exclusive use of H, and possibly preceded by exclusive use of L. When both fuels are used simultaneously, $Z = \bar{Z}$ and $p(t) = \lambda_H(t) + \varepsilon_H \tau(t) = \lambda_L(t) + \varepsilon_L \tau(t)$, so that $\hat{\lambda}_L = \hat{\lambda}_H = \hat{\tau} = \hat{p} = \rho$. Simultaneous use and a binding cap requires that, given a path $R(t)$ of total extraction, $R_L(t) = R_L^z(R(t))$. A price growing at rate ρ implies that $R(t)$ coincides with an unconstrained path, i.e. $R(t) = \tilde{R}(t,\sigma)$ for some σ. Thus, simultaneous use can at most last for the $\log(\bar{p}_L/\bar{p}_H)/\rho$ periods, during which the price grows from $\bar{p}_L$ to $\bar{p}_H$ and cumulative extraction is $\int_0^{\log(\bar{p}_L/\bar{p}_H)/\rho} R_L^z(\tilde{R}(t,S_L^h))\mathrm{d}t = S_L^m$. If $S_{L0} > S_L^m$, simultaneous use must be preceded by exclusive use of L (at $\bar{R}_L$ for $(S_{L0} - S_L^m)/\bar{R}_L$ periods) to allow full exploitation of L. We can now calculate the minimum initial amount of H, given S_{L0}, needed to make the path derived above feasible. The minimum amount is used when the period of exclusive use of H in the constrained period is minimized, which requires τ approaching 0 at the time L is depleted, which, in turn, requires τ approaching 0 when simultaneous use starts. But then, the use of H is, by construction, equal to $\widetilde{S}_H(S_{L0})$, since this was calculated for exclusive H use between 0 and T and a zero tax after exclusive use of L.

Hence, we have proven that if $\lambda_L > \lambda_H$ and $S_{L0} > S_L^m$, we must have $S_{H0} > \widetilde{S}_H(S_{L0})$. Thus, if $\mathbf{S_0} \notin \mathbb{H}$, $S_{L0} > S_L^m$ and $S_{H0} < \widetilde{S}_H(S_{L0})$, we cannot have $\lambda_L > \lambda_H$, and, by lemma 3, we must have equal λ's. Hence, we have proven that if we are in zone I or II, we must have equal scarcity rents. □

Corollary C.1. *Any optimal constrained path outside of zones I and II, entails* $\lambda_L(t) > \lambda_H(t)\ \forall t \in [0,\infty)$.

We now have all the necessary elements to prove that the optimal paths look indeed like they are drawn in Figure 3.

C.1 Endowments in zone I

For initial endowments in the interior of zone I, $\lambda_H(t) = \lambda_L(t)$, and optimal paths enter zone $OABC$ crossing the iso-energy line BC, or hit the abscissa, having exhausted H prior to enforcement. Both in the interim phase, and after the constraint becomes slack, extraction is a matter of indifference. When the constraint binds, however, $\tau(t) > 0$ and, from (4), $\lambda_L(t) + \varepsilon_L\tau(t) < \lambda_H(t) + \varepsilon_H\tau(t)$, thus implying that only L is used in the constrained phase. Hence, the ordering of extraction is indeterminate $\forall t \in [0,T) \cup [T_H,\infty)$, while only L is used in the constrained phase.

The path of the price $p(t)$ over time can is presented in Figure C.1, where the solid line represents the actual path of the price of energy.

C.2 Endowments in zone II

The path is identical to the previous case, with the notable exception that now the trajectory must enter zone I before the enforcement phase begins at time T to ensure that the path enters zone $OABC$ crossing the iso-energy line BC, or hit the abscissa, having exhausted H prior to enforcement. This is required since in the proof of lemma C.1 we show that, we must have $S_L(T_H) \in [S_L^m, S_L^h]$, and $S_H(T_H) \in [0, S_H^m]$, where T_H is the time the constraint becomes slack. In terms of the evolution of the price over time, the graph is qualitatively identical to the one drawn for endowments in zone I (see Figure C.1).

C.3 Endowments in zone IV

Paths starting from zone IV, $\lambda_H(t) < \lambda_L(t)$. Thus, it is optimal to extract only H $\forall t \in [0,T) \cup [T_H,\infty)$. Since $S_{L0} < S_L^m$, the phase of simultaneous extraction cannot be preceded by exclusive use of L (see proof of lemma C.1). Hence, the constrained period begins with an initial phase of joint extraction at the cap along a trajectory parallel to AB, until L is exhausted. The path is parallel to AB, since path AB represents extraction at the cap with simultaneous use and the price, p, growing at rate ρ; these characteristics both apply to an unconstrained path with maximum H stock (i.e. along AB) and to a constrained path with simultaneous use and unequal λ's (see proof of lemma C.1).

L is exhausted as the price reaches $\bar{p}_H$, at time T_{lh} say. Following this, only H is extracted at the cap until the cap ceases to bind – at point A – and the rest of the path is a pure Hotelling one to exhaustion.

The price path corresponding to this extraction path is presented in Figure C.2.

C.4 Endowments in zone III

For endowments in zone III, $\lambda_H(t) < \lambda_L(t)$. Thus, in the interim phase only H is extracted. Since $S_{L0} > S_L^m$, the phase of simultaneous extraction must be preceded by exclusive use of L (see proof of lemma C.1). Once $S_{L0} = S_L^m$, i.e. once zone IV (or area $AA'B'B$) is reached from the right, extraction evolves along a path parallel to AB, until L is depleted, as already explained for zone IV. The last part of the constrained period entails extraction of only H at the cap, until the constraint turns slack at point A. The last part is a pure Hotelling path until exhaustion.

Figure C.3 details the price path of energy when initial endowments are in zone III.

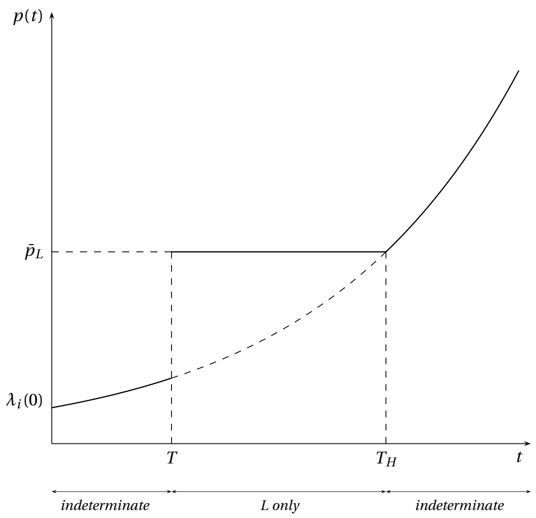

Figure C.1: *Price paths for endowments in zones I and II.*

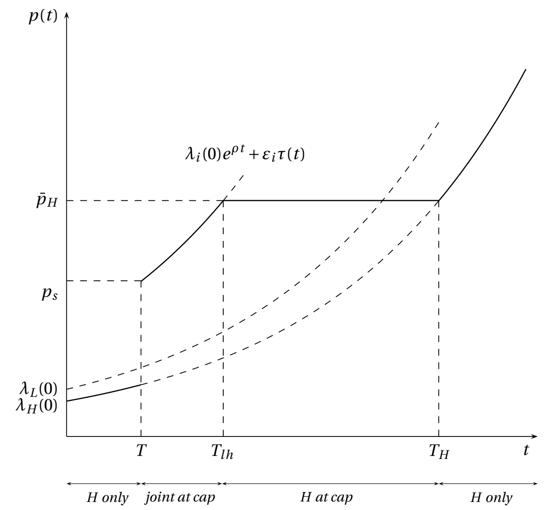

Figure C.2: *Price paths for endowments in zone IV.*

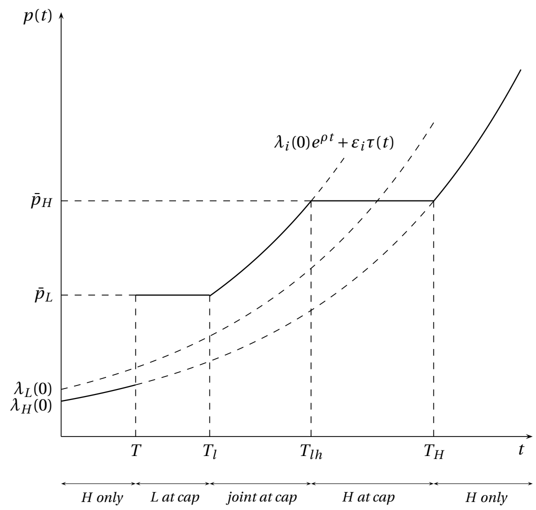

Figure C.3: *Price paths for endowments in zone III.*