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Hierarchical Auctions and Multilevel Mediation in Two-Sided Markets

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Abstract The flourishing internet-based trade motivates our model of two-sided markets with multilevel mediation, in which a seller, intermediaries at various levels and buyers are embedded in a symmetric tree. We extend the framework of the single-unit first price auction to trees and calculate explicitly the equilibrium bids and payoffs to all players in a tree. We show, in particular, the irrelevance of the tree structure for the buyers (leafs in the tree) and order symmetric trees with respect to the payoff to the seller (root of the tree). Furthermore, we characterize stable mediation structures in the presence of link (search) costs. JEL classification: C78, D44, D82.

Keywords two-sided markets, auctions, search engines, internet

1 Introduction

The advent of the internet has seen a widespread use of virtual mediation. Henshaw (2001) reports that 80% of internet users employ search engines to locate information. For example, search engines are used to obtain an instantaneous selection of offers from various providers operating in the cyberspace. Internet auctions, on the other hand, connect potential sellers and buyers from different locations in a virtual auction. Both forms of electronic mediation are facilitated by cybermediaries - business organizations that use internet-based technology to process transactions between buyers and sellers. As the internet dramatically decreased the cost of information search, many had come to believe that traditional intermediaries would become an endangered species in

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electronic markets. In the IT industry, Gates (2000) was amongst many who predicted that disintermediation would be a key feature of the evolving digital economy. In academia, Benjamin and Wigand (1995), Strader and Shaw (1997) and many others pointed out the financial incentives to "cut out the middle man" created by the internet. However, the inevitability of disintermediation has been also questioned with Sarkar et al. (1996) and Chircu and Kauffmann (1999) amongst those who predicted that a new generation of cybermediaries would emerge in a process of "re-intermediation".

The aim of this paper is to contribute to the theoretical literature on intermediaries (market makers, platforms) in two-sided markets. The early research in this topic focused on markets with a single intermediary (e.g., Gehrig, 1993 and Yavas, 1996) and was later extended to competing market makers by, e.g., Fingleton (1997), Rust and Hall (2003) and Armstrong and Wright (2007). The role of intermediation has been also analyzed in the context of specific markets: credit card markets (e.g., Rochet and Tirole, 2002; Wright, 2003), software and internet (e.g., Baye and Morgan, 2001; Caillaud and Jullien, 2001; Hagiu, 2006), B2B markets (e.g., Belleflamme and Toulemonde, 2004) and advertising (e.g., Dukes and Gal-Or, 2003).

In this work, we address the issues of (cyber)mediation in a stylized setup of a multilayer auction that unfolds on a symmetric tree. Different auction formats have been studied thoroughly in the traditional literature on mechanism design (e.g., McAfee et al., 1987; Milgrom, 1989; Myerson, 1981) and in the context of electronic markets (e.g., Stahl, 2002; Mathews and Katzman, 2006). We adapt the classical framework of the single-unit first price auction (FPA) to multi-level hierarchies. Specifically, we consider a single seller, who wants to sell an object to one of the potential buyers that privately know their respective valuations of this object. All our results hold also in the symmetric case of a procurement auction, in which a single buyer wants to buy a homogeneous good from one of the several sellers that privately know their production costs. In either case, the two sides of the market might be unable to communicate and trade directly and depend on intermediaries to transmit the trade-related information. A novel aspect of our model is the hierarchical structure of mediation: The information transmission takes place in a symmetric tree with possibly many layers of intermediaries. At the bottom of the tree, potential buyers with idiosyncratic valuations of the object submit offers to their connected intermediaries. The latter compute the resale value of the object from the collected bids and submit their offers to intermediaries at the next higher level. This process propagates to the highest level, where bids are submitted directly to the single seller situated at the top of this virtual pyramid. The tree structure implies that each bidder submits only one bid to a single connected agent at the next higher level. We think of these structures as virtual and dynamic in nature: An auction tree comes into being when an agent wants to sell an object and it is, typically, dismantled after the transaction. Therefore, there may be many potential sellers in the market and several auction trees operating at the same time. For example, price comparison sites gather offers and present a list of quotes to an inquirer whenever they receive a request.

Note that the fact that a list and not a single price is quoted is inconsequential as the inquirer will choose the best offer for a homogeneous good.

In the conventional FPA, suppliers bring their products to the auction place, where buyers also need to be present. Auctioned goods are handed over to buyers, i.e., product flows are coupled with the auction process. We focus, however, on info-mediation in the FPA ignoring the transfer of goods. The profit maximizing intermediaries will transmit modified buyers' bids to the seller but will not be involved in the physical delivery. The auction process will be, therefore, decoupled from the product flow as discussed, e.g., in Gaudeul and Jullien (2005). Consequently, we shall use sometimes the term infomediaries or cybermediaries instead of intermediaries.

Our results show that the total expected payoff in a hierarchical auction is the same as in the traditional FPA with the same number of buyers and that the expected payoffs to the buyers (leafs in the tree) are independent of the tree structure. The payoff to the single seller (root of the tree), on the other hand, reflects the topology of the tree. We order symmetric trees according to the payoffs received by the seller. On the top of this ordering is the flat tree, in which the seller is connected directly to all buyers. This confirms the well-known result that in any conceivable auction mechanism, seller can not expect more than the FPA payoff. Although seller's preference over auction trees may change in the presence of link (search) costs, we show that at most one level of intermediation can be sustained in a strongly stable tree. Large mediated trees, however, are fragile and are unlikely to be stable under the (myopic) concept of strong stability and also when agents are farsighted. Farsighted stability allows, however, for a wide range of plausible mediation structures. Generally, disintermediation seems to be a robust phenomenon that is more related to the market size and fixed participation costs than to a technology-induced decrease of the search costs or cost differentiation.

In order to analyze hierarchical mediation, we need to specify the bidding behavior of buyers and profit seeking cybermediaries at various levels. Under certain circumstances, the bid function in the standard FPA can be adapted to mediated auctions. For the sake of completeness, we derive this function in the next section and adapt it to symmetric hierarchies in Section 3. In Section 4, we discusses our main results. Section 5 concludes.

2 First Price Seller Auction

In the standard FPA, a seller possesses an object that has an intrinsic value for each of the potential buyers (the value of the object for the seller is normalized to zero). The valuation of the good is private information but all players know the distribution from which it is drawn. Assuming that all valuations are i.i.d. according to the cumulative distribution function (cdf) $F_m^{\overline{v}}(v) := (v/\overline{v})^m$, $v \in [0, \overline{v}]$, we shall find a symmetric bid function $q(v)$ of a buyer with valuation v. Note that $F_m^{\overline{v}}(v)$ is the cdf of the maximum of m uniform random variables, each with the support $[0, \overline{v}]$. In particular, $F_1^1(v) = v$ is the standard uniform distribution.

We denote by $u(v, q)$ the expected payoff of a risk neutral buyer with the valuation (type) v, when she bids q. If n buyers participate in the FPA and bid according to the monotone function $q(.)$ with the inverse $\varphi(.)$, the expected buyer's payoff is equal to

$$u(v, q) = (v - q) F_m^{\overline{v}}(\varphi(q))^{n-1}, \tag{1}$$

where $F_m^{\overline{v}}(\varphi(q))^{n-1}$ is the probability that the buyer with the valuation $v = \varphi(q)$ offers the highest among n bids. The first order condition leads to the differential equation,

$$q' = \frac{F_m'^{\overline{v}}(v)}{F_m^{\overline{v}}(v)}(n-1)(v-q) = \frac{m}{v}(n-1)(v-q). \tag{2}$$

By observing that the lowest type will not offer a positive price, $q(0) = 0$, we obtain as the unique solution to (2) the linear bid function,

$$q(v) = \frac{m(n-1)}{m(n-1)+1} v. \tag{3}$$

Ex-ante, the bid $q(v)$ is distributed according to the cdf $F_m^{q(\overline{v})}$, i.e., as the maximum of m uniform random variables $U[0, q(\overline{v})]$. Therefore, the highest of the n bids (the winning bid) is distributed as the maximum of $m \cdot n$ such variables,

$$(F_m^{q(\overline{v})}(q))^n = (q/q(\overline{v}))^{mn} = F_{mn}^{q(\overline{v})}(q). \tag{4}$$

The distribution (4) of the winning bid allows for the computation of the expected price in the FPA (seller's expected profit),

$$\pi_s = \int_0^{q(\overline{v})} q dF_{mn}^{q(\overline{v})}(q) = \frac{q(\overline{v})mn}{1+mn}. \tag{5}$$

On the other hand, the expected payoff to a buyer in this FPA is obtained from the difference between the price and the winner's valuation,

$$\pi_b = \int_0^{\overline{v}} F_m^{\overline{v}}(v)^{n-1}\{v - q(v)\} dF_m^{\overline{v}}(v) = \tag{6}$$

$$\frac{m\overline{v}}{(1+mn-m)(mn+1)}.$$

As we explain in the next section, properly parametrized formulae (3), (5) and (6) define the bid functions and players' payoffs, respectively, in hierarchical auctions.

3 Hierarchical First Price Auctions in Symmetric Trees

We shall model a hierarchically mediated auction as unfolding in a (directed) symmetric tree (ST). Formally, a ST $\mathbf{k} := (k_1, ..., k_K)$ is a sequence of K branching factors. For each node at level $h = 1, .., K$, $k_h \geq 1$ specifies the number of its child nodes (its span of control). This node is connected to the child nodes by k_h outgoing links and, for $h > 1$, to the parent node by a single incoming link. We denote a subtree $(k_i, ..., k_j)$ of $\mathbf{k}$ by $\mathbf{k}_{i:j}$ and abbreviate $\mathbf{k}_{i:j}$ to $\mathbf{k}_{i:}$ if $j = K$. For notational convenience, we set $\mathbf{k}_{i:j} := \mathbf{k}_{1:j}$ if $i < 1$, $\mathbf{k}_{i:j} := \mathbf{k}_{i:}$ if $j > K$ and $\mathbf{k}_{i:j} := (1)$ if $i > j$. The number of leafs in the subtree $\mathbf{k}_{i:j}$ (in particular, in the tree $\mathbf{k} = \mathbf{k}_{1:K}$) is computed as $\langle \mathbf{k}_{i:j} \rangle := k_i \times .... \times k_j$. We will repeatedly make use of the identity $\langle \mathbf{k} \rangle = \langle \mathbf{k}_{1:h-1} \rangle \langle \mathbf{k}_{h:} \rangle$, where $\langle \mathbf{k}_{1:h-1} \rangle$ is the number of nodes at level h in $\mathbf{k}$ and $\langle \mathbf{k}_{h:} \rangle$ is the number of leafs in each subtree emanating from a node at level h. The simplest ST is the flat tree $\mathbf{f} := (f_1)$ with a single root connected to f_1 leafs. In what follows, we restrict our attention to non-degenerated STs with $k_h \geq 2$ at each level h.

In the context of mediated two-sided markets, we will refer to a ST as an auction tree (AT). Unlike a generic ST, the AT $\mathbf{k} := (k_1, ..., k_K)$ has a specific type of nodes at each level: A seller at level one (the root of $\mathbf{k}$) that is connected to k_1 second-level infomediaries, each of whom in turn connected to k_2 third-level infomediaries etc. At level K, each of the $k_1 \times ... \times k_{K-1}$ intermediaries is linked to k_K buyers (leafs of $\mathbf{k}$). A node at level h will be called agent or player h. Links between nodes at two adjacent levels represent communication channels that transmit trade related information.

An instance of the AT $\mathbf{k}$ is created whenever a seller wants to sell an object. First, the seller establishes k_1 links to intermediaries at level 2. Each of the latter contacts k_2 players at level 3 etc. Finally, each of the level K intermediaries asks k_K buyers to submit bids. The structure $\mathbf{k}$ is common knowledge for all involved players but not necessary the identities of its nodes. With the structure $\mathbf{k}$ in place, the hierarchical FPA (HFPA hereafter) starts at the bottom level $K+1$, where each buyer draws a private valuation υ_{K+1} of the object from the standard uniform distribution, $\upsilon_{K+1} \sim F_1^1(\upsilon) = \upsilon$. Upon learning her type, each buyer submits a bid competing with $\langle \mathbf{k} \rangle - 1$ other buyers and rationally expecting that all agents, located at the same level, use the same monotonic bid function. As this, together with the symmetry of $\mathbf{k}$, implies that the highest offer wins the auction, buyers bid according to (3) with $n = \langle \mathbf{k} \rangle$ and $m = 1$,

$$q_{K+1}(\upsilon_{K+1}) = \upsilon_{K+1} - \upsilon_{K+1}/\langle \mathbf{k} \rangle, \quad \upsilon_{K+1} \sim F_1^1. \tag{7}$$

From (7) follows that bids are ex-ante $F_1^{\overline{\upsilon}_K}$-distributed, i.e., uniformly on $[0, \overline{\upsilon}_K]$, where $\overline{\upsilon}_K = q_{K+1}(\overline{\upsilon}_{K+1} = 1)$. Each intermediary at level K receives k_K independently distributed bids and the highest one is the price, at which this agent can resell the object to the winning buyer. We assume that the object has no intrinsic value for the cybermediaries. Consequently, we identify the valuation (type) υ_K of a cybermediary K with the resale value (the highest received bid). It follows that υ_K is $F_{k_K}^{\overline{\upsilon}_K}$-distributed, i.e., as a maximum of k_K bids, where each bid is drawn from $U[0, \overline{\upsilon}_K]$.

In general, an intermediary $h = 2, ..., K$, observes k_h bids from the connected players at level $h+1$. As an inductive hypothesis, we assume that each of these k_h bids is distributed according to the cdf $F_{\langle \mathbf{k}_{h+1:} \rangle}^{\overline{\upsilon}_h}$, where $\langle \mathbf{k}_{h+1:} \rangle$ is the number of buyers in the subtree rooted in node $h+1$, $\overline{\upsilon}_h = q_{h+1}(\overline{\upsilon}_{h+1})$ is the highest possible bid of node $h+1$ and the bid function q_{h+1} is defined below. We identify the type υ_h of an intermediary h with the maximum of the k_h received offers (i.e., with the resale value). Hence, υ_h is distributed as

$$\upsilon_h \sim (F_{\langle \mathbf{k}_{h+1:} \rangle}^{\overline{\upsilon}_h})^{k_h} = F_{k_h \langle \mathbf{k}_{h+1:} \rangle}^{\overline{\upsilon}_h} = F_{\langle \mathbf{k}_{h:} \rangle}^{\overline{\upsilon}_h}.$$

After learning her type, this player competes with $\langle \mathbf{k}_{1:h-1} \rangle - 1$ other agents at level h with the same type distribution and rationally expecting that all players at the same level use identical bid functions. Hence, she bids according to (3) with $n = \langle \mathbf{k}_{1:h-1} \rangle$ and $m = \langle \mathbf{k}_{h:} \rangle$,

$$q_h(\upsilon_h) = \frac{\langle \mathbf{k} \rangle - \langle \mathbf{k}_{h:} \rangle}{\langle \mathbf{k} \rangle - \langle \mathbf{k}_{h:} \rangle + 1} \upsilon_h. \tag{8}$$

The bid that player h submits to the connected player $h-1$ is, therefore, distributed according to,

$$q_h(F_{\langle \mathbf{k}_{h:} \rangle}^{\overline{\upsilon}_h}) = F_{\langle \mathbf{k}_{h:} \rangle}^{q_h(\overline{\upsilon}_h)} = F_{\langle \mathbf{k}_{h:} \rangle}^{\overline{\upsilon}_{h-1}},$$

which confirms our inductive hypothesis on the distribution of bids, received by player h. The fact that the bid function (8) does not depend on the identity of the bidder corroborates, furthermore, the equilibrium expectation of equal bidding behavior of all agents at the same level. It implies also that the HFPA is an efficient mechanism that assigns the object to the buyer with the highest valuation.

The winning buyer at level $K+1$ pays the bid to the single connected intermediary K, who in turn pays the price that she offered to "her" intermediary $K-1$ etc. The mark-up of a wining intermediary is computed, therefore, as the difference between the price she receives (her valuation) and the price she pays (her bid). Finally, the winning agent at level 2 pays her bid to the seller (as we mentioned in the introduction, we ignore here the physical part of all transactions).

After substituting $\langle \mathbf{k}_{1:h-1} \rangle = n$, $\langle \mathbf{k}_{h:} \rangle = m$, $\overline{\upsilon}_h = \overline{\upsilon}$ and $\overline{\upsilon}_h = q(\overline{\upsilon}_{h+1})$ into (5) and (6), we obtain the expected payoff to an agent h (buyer, seller or infomediary) in the AT $\mathbf{k} = (k_1, ..., k_K)$,

$$\pi_h(\mathbf{k}) = \frac{\langle \mathbf{k}_{h:} \rangle \overline{\upsilon}_h}{(\langle \mathbf{k} \rangle - \langle \mathbf{k}_{h:} \rangle + 1)(\langle \mathbf{k} \rangle + 1)}, \quad h = 1, ..., K+1, \tag{9}$$

where,

$$\begin{aligned}\overline{v}_h = \overline{v}_h(\mathbf{k}) = q_{h+1}(\overline{v}_{h+1}) &= \frac{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{h+1:}\rangle}{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{h+1:}\rangle + 1}\overline{v}_{h+1} \\ &= \Pi_{i=h+1}^{K+1}\frac{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{i:}\rangle}{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{i:}\rangle + 1},\end{aligned} \tag{10}$$

with the convention $\Pi_{i=x}^{y} = 1$ when $x > y$. The expected payoff (9) is strictly positive and depends on the level h, except for the buyers where it simplifies to,

$$\pi_{K+1}(\mathbf{k}) = \frac{1}{\langle\mathbf{k}\rangle(\langle\mathbf{k}\rangle + 1)}. \tag{11}$$

The latter players are, therefore, indifferent between distinct tree structures with the same number $\langle\mathbf{k}\rangle$ of buyers-leafs. From (11), we can also derive the ratio of total expected payoffs at adjacent levels,

$$\frac{\langle\mathbf{k}_{1:h-1}\rangle\pi_h(\mathbf{k})}{\langle\mathbf{k}_{1:h}\rangle\pi_{h+1}(\mathbf{k})} = \frac{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{h+1:}\rangle}{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{h:}\rangle + 1}, \quad h = 1, ..., K. \tag{12}$$

This ratio is weakly greater than one for any $\mathbf{k}$ and level h. Hence, the total payoffs decrease weakly and the individual payoffs decrease strictly as one moves down the hierarchy. In particular, the ratio (12) is equal to $(k_1 - 1)\langle\mathbf{k}_{2:}\rangle$ for $h = 1$. Since the selling mechanism is efficient, the total expected payoff in $\langle\mathbf{k}\rangle$ must be equal to,

$$\begin{aligned}\Pi(\mathbf{k}) &:= \Sigma_{h=1}^{K+1}\langle\mathbf{k}_{1:h-1}\rangle\pi_h(\mathbf{k}) \\ &= \langle\mathbf{k}\rangle/(\langle\mathbf{k}\rangle + 1) = E[F^1_{\langle\mathbf{k}\rangle}],\end{aligned} \tag{13}$$

where $E[F^1_{\langle\mathbf{k}\rangle}]$ is the expected maximum of $\langle\mathbf{k}\rangle$ standard uniform random variables. Therefore, the total payoff in the HFPA does not depend on the tree structure and is equal to the total payoff in the standard FPA (flat tree) with the same number of bidders.

4 Results

First, we analyze the impact of the tree topology on the expected payoffs to player-nodes in an AT. As the buyers' payoffs (11) are not affected by the tree structure, we focus on the expected payoffs to the seller and intermediaries. The following concept of tree dominance will prove useful for the comparison of seller's profits. We shall say that the ST $\mathbf{k} := (k_1, ..., k_K)$ weakly dominates ($\succeq$) the ST $\mathbf{t} := (t_1, ..., t_T)$ if the number of nodes at each level is weakly greater in $\mathbf{k}$ than in $\mathbf{t}$,

$$\mathbf{k} \succeq \mathbf{t} \Leftrightarrow \langle\mathbf{k}_{1:h}\rangle \geq \langle\mathbf{t}_{1:h}\rangle, \quad h = 1, ..., \max(T, K). \tag{14}$$

The dominance is strict ($\succ$) if at least one of the inequalities is strict. Note that $\mathbf{k}$ and $\mathbf{t}$ can have different heights. For example, the following trees with the same number of leafs are completely ordered with respect to the relation (14),

$$(64) \succ (8,8) \succ (8,4,2) \succ (4,8,2) \succ (4,2,8).$$

In particular, $\mathbf{k} = (8,8) \succ (8,4,2) = \mathbf{t}$ follows from,

$$\langle \mathbf{k}_{1:1} \rangle = \langle \mathbf{t}_{1:1} \rangle = 8, \quad \langle \mathbf{k}_{1:2} \rangle = 64 > \langle \mathbf{t}_{1:2} \rangle = 32,$$
$$\langle \mathbf{k}_{1:3} \rangle = \langle \mathbf{t}_{1:3} \rangle = 64.$$

Our next proposition states that the tree dominance implies the payoff dominance for the seller.

Proposition 1 *For any ATs* $\mathbf{k}$ *and* $\mathbf{t}$,

$$\mathbf{k} \succeq (\succ)\mathbf{t} \Rightarrow \pi_1(\mathbf{k}) \geq (>)\pi_1(\mathbf{t}). \tag{15}$$

Proof: All proofs are relegated to the Appendix.

From the proposition follows that the seller prefers short paths to other players. In particular, she prefers a larger span of control to a smaller one for a given set of intermediaries and buyers. A direct FPA in the flat tree maximizes, therefore, seller's payoff over all HFPAs with the same number of buyers. This is simply an instance of the well-known result that the FPA maximizes seller's profit among all conceivable auction mechanisms.[1] The next result shows that the incentive to "cut out" the middlemen is not limited to the seller.

Proposition 2 *Given the AT* $\mathbf{k} = (k_1, ..., k_K)$*, we define the AT* $\kappa := (k_1, ..., k_H \times k_{H+1}, .., k_K)$*,* $1 \leq H < K$*, by merging the levels* H *and* $H+1$ *in* $\mathbf{k}$*. Then,*

$$\begin{aligned} (i) &\quad \Pi(\kappa) = \Pi(\mathbf{k}), \\ (ii) &\quad \pi_h(\kappa) > \pi_h(\mathbf{k}), \quad \forall h = 1, ..., H, \\ (iii) &\quad \pi_h(\kappa) = \pi_{h+1}(\mathbf{k}), \quad \forall h = H+1, ..., K-1. \end{aligned} \tag{16}$$

By cutting k_H links to the middlemen at level $H+1$ and connecting instead to the $k_H k_{H+1}$ nodes at level $H+2$ in $\mathbf{k}$, players at level H will obtain higher payoffs but will not change the payoffs of any agent below H in the merged tree. The latter players have no incentives, therefore, to object to this operation.

The above propositions suggest that a multilevel AT is unstable, as there exists a coalition of players that can "rewire" it and (weakly) improve the payoffs of all its members. This conclusion may be affected, however, when we account for frictions in establishing links. Seller and cybermediaries, for example, may incur a cost if they want to attract prospective bidders.[2]

In the presence of link costs, there is a tension between the benefits from disintermediation, as shown in the above propositions, and the additional cost from the increased number of connections. For example, if the seller pays c for

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[1] This holds under a mild regularity condition (Myerson, 1981, p. 66) which is satisfied in our setup.

[2] Crémer et al. (2007) give several reasons for costly communication in auctions.

each of her outgoing links, she will prefer the mediated tree $(2,2)$ to the flat tree (4) with the same set of buyers if,

$$\pi_1((2,2)) - 2c > \pi_1((4)) - 4c \Leftrightarrow c > 1/10.$$

On the other hand, each intermediary in the mediated tree, that pays the same cost c for an outgoing link, obtains the net profit,

$$\pi_2((2,2)) - 2c = 1/10 - 2c,$$

which is negative when $c > 1/20$. Therefore, for any value of c, either the seller or the intermediaries or both parties will be better off in the flat tree, while the involved buyers are indifferent between the mediated and the flat structure.

Generally, in order to determine stable ATs in the presence of link costs, one needs to specify a set of rules, according to which a network can be modified. The following notion of coalitional deviation can be found, e.g., in Jackson and van den Nouweland (2005).

Definition 1 A network g' is obtainable from g via a deviation by a coalition of players S if,

$$\begin{aligned} &(i) \quad ij \in g' \quad \textit{and} \quad ij \notin g \Rightarrow ij \subseteq S \\ &(ii) \quad ij \in g \quad \textit{and} \quad ij \notin g' \Rightarrow ij \cap S \neq \emptyset. \end{aligned}$$

The condition (i) requires that a new link between players i and j can only be added if i and j belong to S. This reflects the fact that consent of both players is needed to add a link. The condition (ii) requires that at least one player of a deleted link ij be in S. This reflects that fact that either player in a link can unilaterally sever the relationship.

Network stability is assessed with respect to an allocation rule Y that describes how the value of a network g is distributed among the players. For the HFPA game, the allocation rule follows naturally from the payoffs (9) and the link costs,

$$\begin{aligned} &Y_i(g) = \pi_{h(i)}(g) - \rho_i(k_i(g)), \\ &\qquad \text{for agent } i \text{ at level } h(i) \text{ if } g \text{ is an AT}, \\ &Y_i(g) = -\rho_i(k_i(g)), \quad \text{otherwise}, \end{aligned} \tag{17}$$

where the (weakly) increasing function $\rho_i(k_i(g))$, $\rho_i(0) = 0$, determines the total cost for node i of its $k_i(g)$ outgoing links in the (directed) graph g. The value of a network g is then, simply, the sum of $Y_i(g)$ over all players (nodes) in g. This value is equal to the total cost of links in g if g is not an AT as the HFPA equilibrium payoffs (or the HFPA itself) may, then, be undefined.

The HFPA game, with the above set of rules for changing a network, payoffs (17) and coalitions that can be formed by any subset of players, that are present in the market, will be called endogeneous hierarchical auction (EHA) game.

First, we characterize ATs in the EHA game that are strongly stable in the sense of Dutta and Mutuswami (1997). Their definition requires that no coalition can deviate and strictly improve the payoffs of all its members.

Definition 2 Strong stability (Dutta and Mutuswami, 1997): Network g is strongly stable with respect to the allocation rule Y if there is no coalition of players S and g' that is obtainable from g via deviations by S, such that $Y_i(g') > Y_i(g)$ for all $i \in S$.

A strongly stable network corresponds to a strong Nash equilibrium of a network game, in which a player's strategy consists of the set of players, with whom she wants to form a link, and a link forms if and only if two players want it to form. Jackson and van den Nouweland (2005) tighten the latter definition by requiring that at least one player in the deviating coalition is better and no player is worse off. This condition is harder to satisfy and leads, therefore, to a more attractive solution, when it exists. Although our next result holds for both definitions of strong stability, we will use this term in the sense of Dutta and Mutuswami (1997) in the remainder of the paper.

Proposition 3 *A strongly stable AT (SSAT) has at most one level of intermediation.*

A multilevel mediation cannot be a feature of a strongly stable network independently of players' cost functions. In the particular case, in which more than two links are prohibitively expensive, an SSAT has only two possible forms, $(2, 2)$ and (2), although a larger total surplus (13) could be created in a multilevel tree $(2, 2, ..., 2)$ for a sufficiently low cost of two links. The EHA game illustrates, therefore, the well-known conflict between (strong) stability and efficiency.[3]

In the proof of the last proposition, we show that the first level intermediaries benefit from cutting out all intermediaries at lower levels, and that their payoffs diminish with the number of buyers they are directly connected to. This implies that a mediated SSAT must be of the form $(k, 2)$. Although the first level intermediaries benefit from limiting the number of buyers that the seller can indirectly access, for some cost functions the latter player will still prefer a mediated tree $(k, 2)$ to a flat tree.

Disintermediation holds, however, for a common linear cost function $\rho_i(k) = \rho(k) = a + ck \geq 0$. In this case, a strongly stable AT $(k, 2)$ satisfies,

$$\pi_1((k, 2)) - \rho(k) \geq \pi_1((2k-1)) - \rho(2k-1). \tag{18}$$

Otherwise, the seller and $2k-1$ out of $2k$ buyers in $(k, 2)$ will be strictly better off by deviating to the flat tree $(2k-1)$. By substituting from (9), one obtains a condition for c,

$$c \geq (k + 2k^2)^{-1} = \pi_2((k, 2)).$$

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[3] Jackson and van den Nouweland (2005) define a network g as efficient if it maximizes the total payoff over all networks that can be formed by N players,

$$g \ \ efficient \Leftrightarrow \Sigma_{i \in N} Y_i(g) \geq \max_{g' \in 2^N} \Sigma_{i \in N} Y_i(g').$$

A mediated AT $(k, 2)$ can be strongly stable only if the link cost c exceeds the payoff $\pi_2((k, 2))$ of an intermediary in this tree. But then, each intermediary earns a negative payoff due to its two buyer links. Therefore, mediated trees cannot be strongly stable for any affine linear costs.

We observe further that SSATs may fail to exist. For $\rho_1(k) = 0.15k$ and $\rho_i(k) = 0$ for $i \neq 1$ and a market with a large number of intermediaries and buyers, the single candidate for an SSAT is $\mathbf{k} = (3, 2)$, as it maximizes the seller's profit over all trees of the form $(k, 2)$. But then, the seller together with intermediaries and buyers, that are not part of $\mathbf{k}$, could deviate to $\mathbf{k}' = (3, 3)$ making all members of $\mathbf{k}'$ strictly better off. The intermediaries in $\mathbf{k}'$ could, however, improve by reverting to $\mathbf{k}$.[4]

While strongly stable trees may fail to exist, the set of plausible ATs can be expanded by allowing players to be farsighted. In particular, Chwe's (1997) consistent sets allow for other mediation structures by taking "farsightedness" fully into account. Similar insights could be obtained by applying also different concepts of farsighted stability (e.g., the bargaining set in Aumann and Maschler, 1964 or the set of groupwise farsightedly stable networks in Grandjean et al., 2010). In a consistent set C, all coalitional deviations from an outcome $a \in C$ are deterred. A deviation by a coalition S from a to an outcome d is deterred if there exists an outcome $e \in C$ such that $d = e$ or e indirectly dominates d and e is not preferred to a by at least one member of S. The outcome e indirectly dominates d if there is a sequence of coalitional deviations that can move from d to e such that all players in the coalition inducing a deviation are better off in e than in the outcome, which they deviate from. Importantly, this concept of stability is weak in the sense that it rules out an outcome only if it cannot be possibly stable, i.e., if there is no consistent story in which it is stable.

Unlike the set of strongly stable trees, the largest consistent set (LCS) is nonempty in the EHA game (Chwe, 1997, Corollary p. 306). In this game, a coalition can deviate from the network (outcome) g to the network g' if g' is attainable from g via a deviation by S (the attainability in the Definition 1 corresponds to the "effectiveness relations" in Chwe, 1997). It is, then, easy to verify that due to the veto power of the seller, the seller's most preferred AT belongs to a consistent set.

For the sake of illustration, consider $\rho_1(k) = 0$ if $k \leq 2$, $\rho_1(k) = 1$ if $k > 2$ and $\rho_i(k) = 0$ for $i \neq 1$. Then, in a market with two intermediaries and an even number of buyers B, the AT $(2, B/2)$ forms a (singleton) consistent set. To see this, notice that any deviation from $(2, B/2)$ to a tree $\mathbf{d}$ makes the seller worse off and can be deterred by the following course of actions:

(1) The seller deletes all her links in $\mathbf{d}$,

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[4] It is noteworthy that the strong stability in the sense of Jackson and van den Nouweland (2005) is much harder to satisfy in the EHA game than the corresponding concept in Dutta and Mutuswami (1997). As intermediaries and buyers are replaceable, any SSAT by the former standard must be such that all agents are connected. This implies that only (B) and $(B/2, 2)$, where B is the number of buyers in the market, are possible candidates. Moreover, $(B/2, 2)$ can be SSAT only if the number of intermediaries is exactly $B/2$.

(2) Subsequently, the grand coalition deviates to $(2, B/2)$.

Farsighted stability allows, therefore, for stable networks with few intermediaries and a large number of buyers - a feature that one would expect, e.g., for price comparison sites. Nevertheless, our next result shows that large mediated trees are not robust even with farsighted agents. First, we introduce the following definition.

Definition 3 The AT $\mathbf{k} = (k_1, .., k_K)$ is individually rational if $\pi_i(\mathbf{k}) \geq \rho_i(k_{h(i)})$ for each player i at level $h(i)$ in $\mathbf{k}$.

Individual rationality rules out negative expected payoffs. It is a mild restriction that nests strong stability, Chwe's (1997) farsighted stability and, arguably, all "reasonable" stability concepts.[5] It allows for a general characterization of plausible mediation structures.

Proposition 4 *A necessary condition for an AT with B buyers to be individually rational is that $\rho_i(2) \leq 2/(B^2 + B)$ for each player $i \neq 1$ in this AT.*

As $\rho_i(2)$ is the a minimum cost that i must bear in an AT, we refer to it as the (fixed) participation cost. Note that buyers do not maintain any outgoing links and, hence, their participation cost is zero. This cost must fall quadratically in B for an intermediary i, in order to ensure a nonnegative payoff to this player. However, cybermediaries' participation costs can hardly be neglected. In the context of modern electronic markets, significant costs are incurred in software production and maintenance as well as in advertising expenses. Although these costs are spread over all instances of ATs, where an intermediary is active, there are still likely to be prohibitively high in trees with many buyers. In this sense, our model suggests that large mediated networks are unlikely to prevail with rational intermediaries.

5 Conclusions

When analyzing multi-level mediation in hierarchical structures, we conceded a very limited role to intermediaries. Essentially, they transmitted information on buyers' valuations to the seller (this information was modified as intermediaries sought to maximize their positional rents). Undoubtedly, the real impact of intermediaries is far richer. In Biglaiser and Friedman (1994), they survive as guarantors of quality, while Sarkar et al. (1996) identify ten roles of cybermediaries and Barnes and Hinton (2007) propose a five-role classification (informational, logistical, transactional, assurance and customization). In our restricted domain of intermediaries' roles, we showed that at most one

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[5] The fact that a strongly stable AT is individually rational is immediate. For the proof that an AT $\mathbf{k}$ in the LCS is individually rational, consider a unilateral deviation by a player i with a negative payoff in $\mathbf{k}$, such that i cuts all his links and obtains the payoff of zero. This deviation cannot be deterred as there is no sequence of (farsighted) improving deviations ending up in a tree $\mathbf{e} \in LCS$, where agent i is linked and obtains a negative payoff.

level of intermediation can be sustained in a strongly stable tree. Large mediated trees are, however, fragile as the intermediaries' payoffs in these trees fall quadratically in the number of buyers. Such trees are unlikely to be stable not only under the (myopic) concept of strong stability but also when agents are farsighted. Farsighted stability allows, however, for a wide range of plausible structures of intermediation. Generally, disintermediation seems to be a robust phenomenon that is more related to the market size and fixed participation costs than to a technology-induced decrease of the search costs or cost differentiation.

6 Appendix

Proof (Proposition 1). Seller's payoffs in $\mathbf{k}$ and $\mathbf{t}$ are computed by (9),

$$\pi_1(\mathbf{k}) = \frac{\overline{v}_1(\mathbf{k})\langle\mathbf{k}\rangle}{\langle\mathbf{k}\rangle + 1}, \quad \pi_1(\mathbf{t}) = \frac{\overline{v}_1(\mathbf{t})\langle\mathbf{t}\rangle}{\langle\mathbf{t}\rangle + 1}.$$

The claim follows because $\mathbf{k} \succeq (\succ)\mathbf{t}$ implies $\langle\mathbf{k}\rangle \geq (>)\langle\mathbf{t}\rangle$ by (14) and $\overline{v}_1(\mathbf{k}) \geq (>)\overline{v}_1(\mathbf{t})$ by the Lemma 1 below.

Proof (Proposition 2) We define $\mathbf{k} = (k_1, ..., k_K)$ and $\kappa := (k_1, ..., k_H \times k_{H+1}, .., k_K)$ for $1 \leq H < K$. (i) $\Pi(\kappa) = \Pi(\mathbf{k})$ follows from (13) as both trees have the same number of buyers, $\langle\kappa\rangle = \langle\mathbf{k}\rangle$. (ii) Since $\langle\kappa\rangle = \langle\mathbf{k}\rangle$ and $\langle\kappa_{h:}\rangle = \langle\mathbf{k}_{h:}\rangle$ for $h = 1, ..., H$, (9) implies that,

$$\frac{\pi_h(\kappa)}{\pi_h(\mathbf{k})} = \frac{\overline{v}_h(\kappa)}{\overline{v}_h(\mathbf{k})}, \quad 1 \leq h \leq H.$$

Hence, $\pi_h(\kappa) > \pi_h(\mathbf{k})$ if $\overline{v}_h(\kappa) > \overline{v}_h(\mathbf{k})$. The latter inequality follows from (10) as $\langle\kappa\rangle = \langle\mathbf{k}\rangle$ and,

$$\begin{aligned}
&\langle\kappa_{h:}\rangle = \langle\mathbf{k}_{h:}\rangle \Rightarrow \frac{\langle\kappa\rangle - \langle\kappa_{h:}\rangle}{\langle\kappa\rangle - \langle\kappa_{h:}\rangle + 1} = \frac{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{h:}\rangle}{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{h:}\rangle + 1}, \\
&\qquad h = 1, ..., H, \\
&\langle\kappa_{h:}\rangle = \langle\mathbf{k}_{h+1:}\rangle = \frac{\langle\mathbf{k}_{h:}\rangle}{k_h} < \langle\mathbf{k}_{h:}\rangle \\
&\qquad \Rightarrow \frac{\langle\kappa\rangle - \langle\kappa_{h:}\rangle}{\langle\kappa\rangle - \langle\kappa_{h:}\rangle + 1} > \frac{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{h:}\rangle}{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{h:}\rangle + 1}, \\
&\qquad h = H + 1, ..., K.
\end{aligned}$$

(iii) $\pi_h(\kappa) = \pi_{h+1}(\mathbf{k})$ for $h = H + 1, ..., K$, follows from (9) as $\langle\kappa\rangle = \langle\mathbf{k}\rangle$, $\langle\kappa_{h:}\rangle = \langle\mathbf{k}_{h+1:}\rangle$ and $\overline{v}_h(\kappa) = \overline{v}_{h+1}(\mathbf{k})$.

Proof (Proposition 3). The structure of a SSAT, $(k,2)$ or (k), follows from Lemma 3 as for any AT $(k_1,...,k_K)$, $K \geq 2$, the k_1 first level intermediaries and $2k_1$ out of the $k_1...k_K$ buyers will be strictly better off in the AT $(k_1,2)$.

Proof (Proposition 4).

A necessary condition for the AT $\mathbf{k}=(k_1,...,k_K)$, $K \geq 2$, to be individually rational follows from (12), Lemma 3 and (9) as,

$$\begin{aligned}\pi_h(\mathbf{k}) \leq \pi_2(\mathbf{k}) \leq \pi_2((k_1,2)) = \frac{1}{k_1(1+2k_1)},\\ h=2,..,K+1.\end{aligned}$$

The cost that each player $h>1$ can bear in the AT $\mathbf{k}$ cannot exceed the payoff $\pi_2((k_1,2)) \geq \pi_h(\mathbf{k})$ in the tree $(k_1,2)$ with $B=2k_1$ buyers.

Lemma 1

$$\mathbf{k} \succeq (\succ)\mathbf{t} \Rightarrow \overline{v}_1(\mathbf{k}) \geq (>)\overline{v}_1(\mathbf{t}).$$

Proof (Lemma 1). We prove the lemma for $\mathbf{k}=(k_1,...,k_K)$, $\mathbf{t}=(t_1,...,t_T)$ and for the weak dominance $(\succeq)$. We consider two cases, $K \leq T$ and $K>T$.

(i) For $K \leq T$, we expand the definition (10) for $\mathbf{k}$ and $\mathbf{t}$, respectively,

$$\begin{aligned}\overline{v}_1(\mathbf{k}) &= \mathit{\Pi}_{h=2}^{K+1}\frac{\langle\mathbf{k}\rangle-\langle\mathbf{k}_{h:}\rangle}{\langle\mathbf{k}\rangle-\langle\mathbf{k}_{h:}\rangle+1},\\ \overline{v}_1(\mathbf{t}) &= \mathit{\Pi}_{h=2}^{K+1}\frac{\langle\mathbf{t}\rangle-\langle\mathbf{t}_{h:}\rangle}{\langle\mathbf{t}\rangle-\langle\mathbf{t}_{h:}\rangle+1}\mathit{\Pi}_{h=K+2}^{T+1}\frac{\langle\mathbf{t}\rangle-\langle\mathbf{t}_{h:}\rangle}{\langle\mathbf{t}\rangle-\langle\mathbf{t}_{h:}\rangle+1}.\end{aligned}$$

As the product between $h=K+2$ and $T+1$ is either one $(K=T)$ or less than one $(K<T)$, $\overline{v}_1(\mathbf{k}) \geq \overline{v}_1(\mathbf{t})$ when the inequalities,

$$\begin{aligned}\frac{\langle\mathbf{k}\rangle-\langle\mathbf{k}_{h:}\rangle}{\langle\mathbf{k}\rangle-\langle\mathbf{k}_{h:}\rangle+1} \geq \frac{\langle\mathbf{t}\rangle-\langle\mathbf{t}_{h:}\rangle}{\langle\mathbf{t}\rangle-\langle\mathbf{t}_{h:}\rangle+1} \Leftrightarrow\\ \langle\mathbf{k}\rangle-\langle\mathbf{k}_{h:}\rangle \geq \langle\mathbf{t}\rangle-\langle\mathbf{t}_{h:}\rangle,\end{aligned}$$

hold for all $h=2,...,K+1$. Given the dominance relation (14),

$$\begin{aligned}\mathbf{k} \succeq \mathbf{t} \Leftrightarrow \langle\mathbf{k}_{1:h-1}\rangle \geq \langle\mathbf{t}_{1:h-1}\rangle \quad \Leftrightarrow\\ \langle\mathbf{k}\rangle/\langle\mathbf{k}_{h:}\rangle \geq \langle\mathbf{t}\rangle/\langle\mathbf{t}_{h:}\rangle, \quad \forall h=1,...,T,\end{aligned}$$

the latter inequalities hold as there do not exist numbers $\langle\mathbf{k}\rangle$, $\langle\mathbf{k}_{h:}\rangle$, $\langle\mathbf{t}\rangle$ and $\langle\mathbf{t}_{h:}\rangle$ that satisfy the simultaneous conditions,

$$\begin{aligned}\langle\mathbf{k}\rangle \geq \langle\mathbf{t}\rangle \geq 1, \quad \langle\mathbf{k}\rangle/\langle\mathbf{k}_{h:}\rangle \geq \langle\mathbf{t}\rangle/\langle\mathbf{t}_{h:}\rangle \geq 1,\\ \langle\mathbf{k}\rangle-\langle\mathbf{k}_{h:}\rangle < \langle\mathbf{t}\rangle-\langle\mathbf{t}_{h:}\rangle.\end{aligned}$$

If such numbers existed, the above conditions would also hold for $\langle\mathbf{k}\rangle$, $\langle\mathbf{k}_{h:}\rangle$, τ and $\langle\mathbf{t}_{h:}\rangle$, where $\tau \geq \langle\mathbf{t}\rangle$ is such that either $\langle\mathbf{k}\rangle = \tau$ and $\langle\mathbf{k}\rangle/\langle\mathbf{k}_{h:}\rangle \geq \tau/\langle\mathbf{t}_{h:}\rangle$ or $\langle\mathbf{k}\rangle > \tau$ and $\langle\mathbf{k}\rangle/\langle\mathbf{k}_{h:}\rangle = \tau/\langle\mathbf{t}_{h:}\rangle$. Either case leads, however, to a contradiction in the above conditions. For example, $\langle\mathbf{k}\rangle = \tau$ implies the contradicting inequalities,

$$1/\langle\mathbf{k}_{h:}\rangle \geq 1/\langle\mathbf{t}_{h:}\rangle, \quad -\langle\mathbf{k}_{h:}\rangle < -\langle\mathbf{t}_{h:}\rangle.$$

(*ii*) For $K > T$, the iterative application of the Lemma 2 below shows that

$$\overline{v}_1(\mathbf{k}) = \overline{v}_1((k_1, .., k_T, .., k_K)) > \overline{v}_1(\kappa),$$
$$where \quad \kappa := (k_1, .., k_T).$$

On the other hand, $\kappa \succeq \mathbf{t}$ because $\mathbf{k} \succeq \mathbf{t}$ implies $\langle\mathbf{k}_{1:h}\rangle = \langle\kappa_{1:h}\rangle \geq \langle\mathbf{t}_{1:h}\rangle$ for all $h = 1, ..., T$. Since κ and $\mathbf{t}$ have the same length T, $\overline{v}_1(\kappa) \geq \overline{v}_1(\mathbf{t})$ follows from part (*i*).

Lemma 2

$$\overline{v}_1((k_1, ..., k_K, n)) > \overline{v}_1((k_1, ..., k_K)),$$
$$K \geq 1, n \geq 2.$$

Proof (Lemma 2). For $\kappa := (k_1, ..., k_K, n)$ and $\mathbf{k} := (k_1, ..., k_K)$,

$$\overline{v}_h(\kappa) = (\varPi_{h=2}^{K+1} \frac{n\langle\mathbf{k}\rangle - n\langle\mathbf{k}_{h:}\rangle}{n\langle\mathbf{k}\rangle - n\langle\mathbf{k}_{h:}\rangle + 1}) \frac{n\langle\mathbf{k}\rangle - 1}{n\langle\mathbf{k}\rangle}$$
$$> \varPi_{h=2}^{K+1} \frac{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{h:}\rangle}{\langle\mathbf{k}\rangle - \langle\mathbf{k}_{h:}\rangle + 1} = \overline{v}_h(\mathbf{k}),$$

We divide both sides of the last inequality by its r.h.s. and factor out the first K terms,

$$(\varPi_{h=2}^{K} \frac{n\langle\mathbf{k}\rangle - n\langle\mathbf{k}_{h:}\rangle + n}{n\langle\mathbf{k}\rangle - n\langle\mathbf{k}_{h:}\rangle + 1}) \varTheta(\mathbf{k}, n) > 1, \quad where$$
$$\varTheta(\mathbf{k}, n) := \frac{n\langle\mathbf{k}\rangle - n}{n\langle\mathbf{k}\rangle - n + 1} \frac{n\langle\mathbf{k}\rangle - 1}{n\langle\mathbf{k}\rangle} \frac{\langle\mathbf{k}\rangle}{\langle\mathbf{k}\rangle - 1}$$

The claim holds as, for $n \geq 2$, the product in parentheses is strictly greater than one and,

$$\varTheta(\mathbf{k}) = \frac{n\langle\mathbf{k}\rangle - 1}{n\langle\mathbf{k}\rangle - n + 1} \geq 1.$$

Lemma 3

$$\pi_2(\mathbf{k}) < \pi_2((k_1, 2)), \quad \mathbf{k} = (k_1, ..., k_K), K \geq 2.$$

Proof (Lemma 3): By (9), $\langle \mathbf{k} \rangle = \langle \mathbf{k}_{1:h-1} \rangle \langle \mathbf{k}_{h:} \rangle$ and the fact that $\overline{v}_h < \overline{v}_{h+1}$ by (10),

$$\begin{aligned}\pi_2(\mathbf{k}) &= \frac{\langle \mathbf{k}_{2:} \rangle \overline{v}_2}{(\langle \mathbf{k} \rangle - \langle \mathbf{k}_{2:} \rangle + 1)(\langle \mathbf{k} \rangle + 1)} \\ &\leq \frac{\langle \mathbf{k}_{2:} \rangle \overline{v}_K}{(k_1 \langle \mathbf{k}_{2:} \rangle - \langle \mathbf{k}_{2:} \rangle + 1)(k_1 \langle \mathbf{k}_{2:} \rangle + 1)} \\ &= \frac{k_1 \langle \mathbf{k}_{2:} \rangle - 1}{k_1 (k_1 \langle \mathbf{k}_{2:} \rangle - \langle \mathbf{k}_{2:} \rangle + 1)(k_1 \langle \mathbf{k}_{2:} \rangle + 1)}.\end{aligned}$$

The derivative w.r.t. $\langle \mathbf{k}_{2:} \rangle$ of the last expression is negative when k_1, $\langle \mathbf{k}_{2:} \rangle \geq 2$. Hence, the last expression attains its strict maximum over $\langle \mathbf{k}_{2:} \rangle \geq 2$ for $\langle \mathbf{k}_{2:} \rangle = 2$,

$$\frac{(2k_1 - 1)}{k_1(2k_1 - 1)(2k_1 + 1)} = \frac{1}{k_1(2k_1 + 1)} = \pi_2((k_1, 2)).$$

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