Optimal Retirement and Saving with Increasing Longevity

David E. Bloom[1], David Canning[1], and Michael Moore[2]

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We develop a simple life cycle optimizing model of retirement and savings. We show that, in theory, higher incomes lead to earlier retirement and higher savings while longer life spans lead to later retirement and lower savings. We calibrate our model using data from the United States and find that the model predicts that over the last century the effect of rising incomes has been twice as large as the effect of the secular rise in life expectancy.

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1. Harvard School of Public Health.
2. Queen's University, Belfast.

Address for correspondence:
667 Huntington Avenue
Boston, MA 02115
dcanning@hsph.harvard.edu

1. Introduction

The improvement in life expectancy and living standards over the last 150 years constitutes a dramatic increase in human welfare. For the world as a whole, life expectancy at birth rose from around 30 years in 1900 to 65 years by 2000 (and is projected to rise to 81 by the end of this century; Lee 2003). These improvements have not only resulted in a large direct gain in welfare (Nordhaus 2003, Becker, Philipson, and Soares, 2005) but have also had a profound influence on economic life-cycle behavior by changing people's time horizons (Hamermesh, 1985).

In this paper we extend the Blanchard-Yaari-Weil model of consumption with finite lives and perfect annuity markets to endogenously determine retirement, as well as the time path of consumption. We use the model to show how optimal retirement and saving decisions respond to changes in the level of wages and life expectancy. Following Auerbach and Kotlikoff, (1987), we explain retirement by rising ill health with age that increases the disutility of labor and reduces the productivity of working.

While health declines with age, rising life spans have been accompanied by improving age-specific health status; that is, we have lives that are both longer and healthier. The ‘compression of morbidity’ hypothesis (Fries, 1980), maintains that the average age at first infirmity, disability, or other morbidity is postponed to such an extent that the period of ill health at the end of life is compressed. Increases in life expectancy in the United States over the last two centuries have indeed been associated with reductions in the age-specific incidence of disease, disability, and morbidity (Costa 2002; Fogel 1994, 1997). This trend is continuing (Crimmins, Saito and Ingegneri, 1997, Freedman, Martin and Schoeni 2002). We model the compression of morbidity by assuming that the age of onset of disability rises proportionately, or more than proportionately, with life expectancy.

Increases in both wages and lifespan have wealth effects in that they enlarge the budget set. They also have incentive and substitution effects by changing the rewards to working and saving.

Our model shows that, under standard assumptions on preferences, a higher level of wages leads to earlier retirement and increasing savings rates. On the other hand an increase in life expectancy leads to an increase the retirement age, but less than proportionately, while reducing savings rates. The idea that an increase in life expectancy reduces savings rates may appear counterintuitive. The rationale for this result is that an increase in life expectancy expands the size of the budget set, which leads to an increase both in leisure time (years of retirement) and consumption. Bloom, Canning, Mansfield, and Moore (2007) analyse a simple version of this model, assuming that utility is logarithmic in consumption and that the mortality rate is constant, independent of age. We generalize the results of that paper by allowing a general concave utility function, and varying age specific mortality rates.

Our savings result differ from those in previous theoretical papers (Chang (1991), Kalemli-Ozcan and Weil (2010)) who argue rising longevity will increase savings rates. The major difference is that we allow for the compression of morbidity, which allows longer working lives, and assume complete annuity markets, which removes the risk that savings are wasted due to early mortality when life expectancy is low.

Our theoretical model predicts countervailing forces on both the retirement age and saving rate from the secular rise in wage levels and life expectancy. We resolve this by calibrating the model using data from the United States over the last century. Over this period we calibrate that the magnitude effect of higher wage levels on retirement and savings has been twice as large as the effect of longer life spans, so that overall our model predicts a movement towards earlier retirement and higher savings rates. The calibrated decline in retirement ages is consistent with the observed evolution of retirement over the last century (Costa 1998a, 1998b). The predicted rise in the cohort savings rates is consistent with the findings of Attanasio (1998)[1].

[1] Attanasio estimates cohort effects, for cohorts born between 1910 and 1959, by controlling for a common age pattern of saving, and finds this result when treating purchases of housing and consumer durables as saving. The

We present our model in section 2, and in section 3 we show how the dynamic programming problem facing agents generates a set of equations that determine the optimal retirement age and consumption profile. In section 4 we use the model to investigate the theoretical effect of increases in healthy lifespan and wages on savings and retirement behavior. Section 5 calibrates the model to US data from the last century and section 6 makes some concluding remarks[2].

2. The Model

Our formal lifecycle model makes a number of simplifying assumptions. We assume that the mortality schedule is exogenous, ignoring the possibility of using consumption and health services to extend longevity (Ehrlich and Chuma 1990), or of a reverse link from labor supply to health status and life expectancy. In the interest of simplicity, we also assume that the life cycle has no period of schooling. Longer life spans may increase the incentive to invest in education (De la Croix and Licandro 1999) though it is really only duration of working life that earns a return to education (Echevarría and Iza 2006) and the education effect depends on the effect of life span on retirement.

We begin with mortality. We assume that there is a family of possible survival schedules indexed by a single variable λ. There is a survival schedule $s(t,\lambda)$ that gives the probability of survival to age t. The mortality rate at time t is

$$m(t,\lambda) = -\frac{\frac{ds}{dt}(t,\lambda)}{s(t,\lambda)} \tag{1}$$

and life expectancy is given by[3]

increase in cohort household saving rates is consistent with declining national savings due to aggregation across cohorts, non-household saving, and the treatment of consumable durables as consumption.

[2] Proofs of propositions and technical details of the calibration exercise are available in a web appendix.

[3] The probability that your age of death is exactly t is the probability of surviving to t times the mortality rate at t. The second equality comes from substituting in for the mortality rate and integrating by parts using the fact that $s(0,\lambda)=1$ and $s(T,\lambda)=0$.

$$z(\lambda) = \int_0^T t\, m(t,\lambda)\, s(t,\lambda) dt = \int_0^T s(t,\lambda) dt \qquad (2)$$

We assume that there is a biological maximum, T, to the length of life (Carnes, Olshansky, and Grahn, 2003). Let us assume that as the index λ increases, mortality at each age falls, so that survival to each age rises, as does overall life expectancy, z, and that this relationship is invertible. Lee and Miller (2001) show using empirical data that in practice the Lee and Carter (1992) method of indexing mortality rates generates a monotonic relationship between the index and the age-specific mortality rates; adopting this method of indexing would therefore be a method of actually constructing the index λ. Assuming a one-to-one mapping from life expectancy to our index of mortality, we can take life expectancy to be a unique identifying index for a mortality schedule. In what follows we therefore use life expectancy at the beginning of working life, z, as our index of the mortality schedule and write mortality[4] as $m(t,\ z)$.

A major innovative feature of our model is the health schedule $h(t,z)$. We assume that health declines with age t. In addition, we postulate that health at age t depends on life expectancy, z. If health is independent of z, it implies that increases in life expectancy are not associated with general health improvements in the form of reductions in morbidity (sickness), which implies that population aging is associated with an increasingly large share of unhealthy people.

However, evidence points in the opposite direction. Not only are people living longer, but age specific disability is also falling. Cutler (2001) attributes reductions in disability at older ages to long-term protective effects of reduced disease exposure in childhood, rising levels of

[4] Note that we do not impose the Lee-Carter functional form; we only specify that there is a one-to-one mapping from mortality schedules to life expectancy so that life expectancy identifies a mortality schedule.

education and socio-economic status, improved health-related behavior such as reductions in smoking, improved medical care, and the use of special aids to reduce disability for a given health state. Manton, Gu, and Lamb (2006) estimate that between 1935 and 1982 life expectancy and (disability-free) active life expectancy at age 65 both moved up in the United States, keeping the proportion of disability-free life after 65 relatively steady at around 73%. However, between 1982 and 1999 life expectancy at age 65 rose from 16.9 to 17.7 years while active life expectancy rose from 12.3 to 13.9 years. This implies a rise in the proportion of disability-free life to over 78%, and a reduction in the absolute time spent in disability from 4.6 to 3.8 years. The compression of morbidity, either absolute or relative, appears to hold in most developed countries (Mor 2005). Although time-series evidence in developing countries is scarce, Mathers et al. (2001) show that across countries health-adjusted life expectancy (each life year weighted by a measure of health status) increases as fast as total life expectancy, implying a rising proportion of healthy to total life expectancy as life expectancy increases.

Although the compression of disability, at least in the recent past, in the United States seems evident, and although disability is a cause of retirement (Gordo 2006), most people retire before the onset of severe disability. Despite the fact that they may still be physically capable of working, moderate ill health not amounting to disability may deter their participation in the labor force. The "dynamic equilibrium" hypothesis suggests the compression of time spent in severe disability may be the result of reducing the transition rate from low or moderate ill health to severe disability. This in turn may result in an increase in the prevalence of less-severe disability states (e.g., Graham et al. 2004).

To examine the effect of the compression of morbidity we begin by taking as our benchmark the case where healthy life span increases proportionately with overall life span. We assume that:

$$h(\rho t, \rho z) = h(t, z) \ \ for \ \ \rho > 0 \tag{3}$$

Note that we assume both age and life expectancy are measured from the start of adult life. If a particular level of health is labeled "disabled," equation (3) implies that the age of onset of this "disabled" health rises proportionately with life expectancy. After examining this case we discuss how our results extend to the case

$$h(\rho t, \rho z) > h(t, z) \ \ for \ \ \rho > 1 \tag{4}$$

which allows for the absolute compression of morbidity.

We denote labor supply at time t by χ_t, which is assumed to lie in the closed interval $[0,1]$. Felicity is assumed to be strongly separable in goods and leisure and is given by

$$u(c(t)) - \chi(t)v(h(t, z)) \tag{5}$$

where $c(t)$ is consumption at age t and $v(h(t, z))$ is the disutility of working given the health state $h(t, z)$. We assume that u is twice differentiable with $u'(c) > 0, u''(c) < 0$ and that the disutility function v satisfies $v'(h) < 0$ so that the disutility of work (and the relative utility of leisure) is higher when health is poorer. Lifetime expected utility is

$$U = \int_0^T e^{-\delta t} s(t, z)\left[u(c(t)) - \chi(t)v(h(t, z))\right]dt \tag{6}$$

where δ is the subjective rate of time preference, and T is the biological maximum life span. The wage earned by a worker with health h is given by

$$w(t, h) = w_1(t) w_2(h(t, z)) \tag{7}$$

The term $w_1(t)$ captures the change in wages over time due to exogenous forces, for example, technical progress, and $w_2(h(t,z))$ captures the fact that worker productivity depends on health, which is a function of age relative to life expectancy. We assume $w_1'(t) \geq 0$ and $w_2'(h) \geq 0$ so that wages are increasing over time due to exogenous forces and are increasing in health. The multiplicative functional form implies that the health effect on log wages is additive, which is consistent with a Mincer wage equation that includes health as a form of human capital as in Schultz (1999). Chang (1991) and Kalemli-Ozcan and Weil (2010) have exogenous age-wage schedules that do not change as life expectancy varies; we allow for an effect of life expectancy on wages through improved health.

Wealth, W, evolves according to

$$\frac{dW}{dt} = \chi(t)w_1(t)w_2(h(t,z)) + \left(m(t,z) + r\right)W(t) - c(t) \tag{8}$$

If the agent works at time t, he or she earns the wage $w_1(t)w_2(h(t,z))$, and this is added to wealth. Consumption, $c(t)$, reduces wealth, while the stock of wealth earns a return, $m(t,z) + r$. We assume that health may affect labor productivity and wages. We assume that wealth can be transferred from one period to another by saving or borrowing from the financial sector. This competitive financial sector can borrow or lend freely at the interest rate *r*. Agents are paid an effective interest rate $m(t,z) + r$ on their savings, which is larger than *r*, to compensate them for the fact that they may die before withdrawing their savings. Similarly, agents who borrow pay the rate $m(t,z) + r$ to compensate the bank for the fact that they may die before repaying their borrowings. This is equivalent to treating all savings as being in the form of annuity purchases, while assuming that all borrowing has to be accompanied by an actuarially fair life insurance contract for the amount of the loan. Provided that a continuum of agents exists, the financial sector can avoid all risk by aggregating over individuals thereby earning zero profits.

The transfer of wealth from those who die to the financial sector exactly compensates deposit-taking institutions for the fact that they pay an interest rate $m(t,z)+r$ on deposits that exceed the risk-free rate *r*, and rules out the need to consider unintended bequests. The budget constraint is:

$$\int_0^T e^{-rt} s(t,z) c(t) dt \leq \int_0^T e^{-rt} s(t,z) \chi(t) w_1(t) w_2(h(t,z)) dt \qquad (9)$$

The control variables for the agent's optimization problem are c and χ. Agents must decide when to work and what their consumption stream should be.[5]

We assume the boundary conditions:

$$\begin{aligned} w_1(0) w_2(h(0,z)) u'(0) &> v(h(0,z)) \\ w_1(T) w_2(h(T,z)) u'(0) &< v(h(T,z)) \end{aligned} \qquad (10)$$

The first boundary condition implies that the disutility of working at age zero is sufficiently low to ensure that the agent always prefers to work at the start of life, rather than live with no consumption. The second boundary condition implies that the disutility of working at the maximum possible age is sufficiently high that the agent always prefers to be retired.

It what follows we will assume that the rate of interest, r, is positive and equals the rate of time preference, δ. Our theory is developed for the case where the growth rate of the exogenous component of wages is moderate. If wage growth is very rapid a retired worker may re-enter the labor market to take advantage of the high wages when old. To rule this out we assume that, for every life expectancy, the rate of growth of the disutility of labor with age exceeds the exogenous component in the rate of growth of wages at each point in time:

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[5] Adding the direct utility of health as an additive term to the utility function does not affect decision making in any way.

$$\frac{\dot{w}_1}{w_1} < \frac{\dot{v}}{v} \tag{11}$$

We show that this condition will imply that retirement occurs only once and is never reversed. In this case we can label the retirement age as R.

3. Optimal Retirement and Consumption Decisions

The Hamiltonian for this problem is

$$\begin{aligned} H &= e^{-\delta t} s(t,z)\left[u(c(t)) - \chi(t)v(h(t,z))\right] \\ &\quad + \phi(t)[\chi(t)w_1(t)w_2(h(t,z)) + \left(m(t,z)+r\right)W(t) - c(t)] \end{aligned} \tag{12}$$

The first-order conditions for a maximum in c and χ are:

$$\dot{\phi} = -\frac{\partial H}{\partial W} = -\phi(t)(r + m(t,z)), \tag{13}$$

$$\frac{\partial H}{\partial c} = e^{-\delta t} s(t,z) u'\left(c(t)\right) - \phi(t) = 0 \text{, and} \tag{14}$$

$$\begin{aligned} \frac{\partial H}{\partial \chi} &= -e^{-\delta t} s(t,z) v(h(t,z)) + \phi(t) w_1(t) w_2(h(t,z)) \geq 0 \text{ when } \chi(t) = 1 \\ \frac{\partial H}{\partial \chi} &= -e^{-\delta t} s(t,z) v(h(t,z)) + \phi(t) w_1(t) w_2(h(t,z)) = 0 \text{ when } \chi(t) \in (0,1) \\ \frac{\partial H}{\partial \chi} &= -e^{-\delta t} s(t,z) v(h(t,z)) + \phi(t) w_1(t) w_2(h(t,z)) \leq 0 \text{ when } \chi(t) = 0 \end{aligned} \tag{15}$$

These conditions can be shown to yield the following[6]:

$$\frac{\dot{c}}{c} = \left(r - \delta\right) \frac{u'\left(c\right)}{-cu''\left(c\right)} \tag{16}$$

$$\begin{aligned} u'(c(t))w_1(t)w_2(h(t,z)) &> v(h(t,z)) \Rightarrow \chi(t) = 1 \\ u'(c(t))w_1(t)w_2(h(t,z)) &= v(h(t,z)) \Rightarrow \chi(t) \in [0,1] \\ u'(c(t))w_1(t)w_2(h(t,z)) &< v(h(t,z)) \Rightarrow \chi(t) = 0 \end{aligned} \tag{17}$$

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[6] Equation (16) follows from equation (1). We suppress the fact that consumption depends on t though this is implicit in equation (16).

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The first condition is standard and arises because the annuities perfectly insure against financial losses due to death. Given our assumption that $r = \delta$, optimal consumption is constant over time. The second condition implies that agents work at time t so long as the utility gain from the consumption purchased by the wage they earn (the marginal utility of consumption times the wage) exceeds the disutility of working.

The concavity of the utility function u implies that the Hamiltonian is concave in the two control variables. Hence these first-order conditions give a global maximum of the Hamiltonian in the control variables, c and χ. The Hamiltonian is jointly concave[7] in the two control variables and the state variable, wealth, and hence satisfies the Mangasarian condition for our set of equations to give a maximum for the dynamic programming problem (see Sydsaeter, Strøm and Berck 2000).

4. Results

Proposition 1 (a) In the optimal plan retirement occurs once and is not reversed. (b) The retirement age and consumption stream that maximize expected lifetime utility are unique.
Proof: see Web Appendix 1.

Note that retirement emerges endogenously from our model. We first prove that any optimal plan has a single and irreversible retirement age. We then prove that this optimal plan is unique. Note that retirement emerges endogenously from our model. In our framework a lack of health effects on the disutility of labor and wages, combined with exogenous wage growth,

[7] The Hessian of second derivatives is negative semi-definite everywhere.

would tend to produce leisure in youth when wages are low, and work when old and wages are high; the health effects are essential for our model to produce retirement.

At the beginning of life the disutility of labor is low and the benefits of working in terms of additional consumption spread over the life cycle are large. As life proceeds, rising disability eventually outweighs the benefits of working, and retirement occurs. It is possible, if wages were rising very rapidly, that re-entry to the labor market might occur; we rule this out by limiting the rate of wage growth to be lower than the rate of increase in disability with age.

Given proposition 1 we can rewrite the budget constraint as:

$$\int_0^T e^{-rt} s(t,z) c(t) dt \leq \int_0^R e^{-rt} s(t,z) w_1(t) w_2(h(t,z)) dt \qquad (18)$$

In what follows we think of the choice variables as the consumption level and the retirement age, with these chosen subject to equation (18). Consumption is steady over the life cycle. Rising wages for young workers can lead to dis-saving when young before saving in middle age in preparation for retirement when old.

Now consider what happens if life expectancy increases. Lee and Goldstein (2003) argue that a natural benchmark for responding to an increase in life expectancy is to adjust the timing of all life course choices proportionately. Bloom, Canning, and Graham (2003) show that in the case of a certain life span, the optimal response to an increase in life span and health is to increase working life proportionately keeping consumption unchanged, provided that interest rates, time preference rates, and exogenous wage growth are all zero. In our framework, however, it is not clear that such a proportional response is even feasible. Consider an initial life expectancy z_0 and let the optimal retirement age be R_0^* and the optimal consumption level be c_0^*. Let the optimal retirement age at the longer life expectancy, z_1, be R_1^* and the optimal consumption level be c_1^*. We require that the old consumption stream is affordable at the new survival schedule provided retirement is postponed in line with life expectancy.[8] We call this the *proportionality condition*:

$$\int_0^T e^{-rt} s(t, z_1) c_0^* dt \leq \int_0^{R_0^* z_1 / z_0} e^{-rt} s(t, z_1) w_1(t) w_2(h(t, z_1)) dt \quad for\ z_1 > z_0 \qquad (19)$$

This proportionality condition may not be satisfied if the extension in life expectancy is due to a rise in the survival schedule concentrated in the period immediately after retirement. However, if the gain in life span comes from a more general reduction in mortality rates that generates survival increases during the working life (which add to labor income) or near the end of life (where consumption is heavily discounted), the additional income generated by proportionately later retirement will be sufficient to keep consumption unchanged.

In practice, this condition is satisfied in historical data and future projections for the United States. In Web Appendix 2 we examine cohort survival schedules for the United States (the survival curve faced by the cohort) for different birth years. We use data from the United States because we have projections on future mortality rates for cohorts born after 1900, while for other countries we only have the observed completed survival schedules for cohorts born before 1900, without projections. Using long run averages for the real interest rate and wage growth, we show that moving from the survival schedule of those born in 1900 to those born in 1925, 1950, 1975, or 2000 (based on historical cohort survival rates and projected rates) with an increase in the retirement age proportional to the increase in life expectancy, would in each case allow consumption to increase. This means that it has been feasible to keep the working life proportional to life expectancy with consumption unchanged.

[8] Note that by proposition 1 we can summarize labor supply by a retirement age. We integrate labor income over time up to retirement to get the value of lifetime earnings.

We begin by giving results for the relative compression of morbidity as set out in equation (3). We will later extend this result to the case in which compression of morbidity is faster than proportional.

Proposition 2: If there is proportional compression of morbidity, and changes in the survival schedule satisfy the proportionality condition, the optimal response to an increase in life expectancy requires consumption to stay the same or increase.
Proof: see Web Appendix 3.

The intuition for this result is that it is possible for the agent to increase working life proportionately but keep consumption unchanged. In general the agent will do better than this. If both leisure and consumption are normal goods the agent will take some of the additional welfare in the form of consumption and some in the form of increased leisure.

Proposition 3: If there is proportional compression of morbidity, and changes in the survival schedule satisfy the proportionality condition, the optimal response to an increase in life expectancy requires the retirement age to rise less than proportionately with life expectancy.
Proof: see Web Appendix 4.

If $h(\rho t, \rho z) > h(t, z)$ rising life expectancy will increase wages and reduce the disutility of labor substantially in old age, potentially increasing the retirement age substantially. A more interesting case is near the border line where $h(\rho t, \rho z) = h(t, z)$ so that the "benchmark" response is a proportional rise in the retirement age keeping consumption unchanged. In this case the wealth effect of a rise in life expectancy will, as we have seen in proposition 2, increase consumption and as shown in proposition 3, decrease the proportion of life spent working.

As well as the possibility that health is rising faster than life expectancy, we have to guard against the possibility that wages are rising so rapidly over time that the substitution effect dominates the wealth effect and workers increase the proportion of life spent working to take advantage of the higher wages later in life.

We now turn to the issue of how retirement and consumption are affected by a change in the exogenous component of the wage schedule. We consider what happens when the levels of wages rises; we think of the wage schedule moving up proportionately at every point in time. We find that what happens depends crucially on the inter-temporal elasticity of substitution, which in our model is the inverse of the coefficient of relative risk aversion because of the assumption of time-separable utility.

Proposition 4: When the level of wages rises the optimal response of the agent is to

(i) keep retirement unchanged and increase consumption proportionally with wages if the utility function has a local inter-temporal elasticity of substitution of unity.

(ii) increase the age of retirement and increase consumption more than proportionally with wages if the utility function has a local inter-temporal elasticity of substitution greater than unity.

(iii) decrease the age of retirement and increase consumption less than proportionally with wages if the utility function has a local inter-temporal elasticity of substitution less than unity.

Proof: see Web Appendix 5.

It is well known that the impact of wages on consumption and the demand for leisure has both income and substitution effects. The pivotal case occurs where there is relative risk aversion that is locally unity. We state this proposition to show that the result applies to our model. The long-term decline in the retirement age, and increase in retirement savings, is consistent with an inter-temporal elasticity of substitution less than unity (which in our model implies a coefficient of relative risk aversion that exceeds one).

We focus on the effects of changes in life expectancy and the level of wages on retirement and saving decisions. Bloom, Canning, and Moore (2004) also investigate the effects of changes in the interest rate, rate of time preference, and the rate of wage growth under more restrictive functional forms for utility and mortality.

5. Calibration

We now perform a calibration exercise. We solve the model numerically using explicit functional forms and plausible values of the parameters. Moving from theoretical approach to a calibration approach allows to relax some of the assumptions that were required to derive our propositions. In particular we no longer assume that the rate interest equals the rate of time preference which allows for rising individual consumption. In the theoretical section we assumed that the disutility of working was related to life expectancy. We now allow age specific morbidity, and the disutility of working, to be related to the mortality rate at that age.

We assume we have iso-elastic utility

$$u(c) = \frac{c^{1-\beta}}{1-\beta} \tag{20}$$

For the calibration we take $\beta = 2$. Cross country and long time series aggregate studies (as opposed to household level studies) do find that labor supply is lower when wages are higher and gives estimates that are close to $\beta = 2$ (Chetty 2006).

Our utility function implies that optimal consumption at each age t satisfies

$$\frac{\dot{c}}{c} = (r-\partial)\frac{u'(c)}{-cu''(c)} = \frac{(r-\partial)}{\beta}, \quad c(t) = c(0)e^{\frac{(r-\partial)}{\beta}t} \qquad (21)$$

The Lee-Carter method of modeling mortality allows a fully flexible relationship between age and mortality rates though this relationship shifts between cohorts with movements in a single underlying variable. This fully flexible age-mortality relationship requires the estimation of a large number of parameters and for the purposes of calibration we use the simpler Gompertz-Makeham survival function $s(t)$ which has the associated mortality schedule $m(t)$ [9]

$$\mathrm{s}(t) = e^{\left\{-\rho t - \frac{a}{b}\left(e^{bt}-1\right)\right\}}, \ \mathrm{m}(t) = \rho + ae^{bt} \qquad (22)$$

Table 1 gives the estimated values of ρ, a, b for mortality schedules for people aged 20 and over in the United States for cohorts born in 1901, 1951 and 1996 based on data and projections from the Berkeley Mortality Database[10]. The values of the parameters ρ, a, b are estimated to fit the empirical mortality schedule using nonlinear least squares. Figure 1 shows the empirical mortality schedules and the fitted values based on our Gompertz-Makeham function estimates for the 1901 birth cohort where we have actual (rather than projected) mortality rates at all ages. Figure 1 shows that Gompertz-Makeham function provides a reasonable fit to the observed data.

We assume that the disutility of labor at each age is proportional to mortality at that age

$$v(t) = dm(t) = d\left(\rho + ae^{bt}\right) \qquad (23)$$

[9] This is consistent with the theory that mortality moves with a single underlying index λ if the parameters of the Gompertz-Makeham can be written as functions of λ, that is $\rho(\lambda), a(\lambda)$ and $b(\lambda)$.

[10] http://demog.berkeley.edu/~bmd

so that reductions in mortality, and increases in life expectancy are associated with lower age specific disutility of working. The parameter d, the weight given to the disutility of working in the utility function, is calibrated so that optimal retirement of the cohort born in 1900 is 65. This gives us a value of d=0.667.

We normalize the real wage to be 100 in 1921 when the cohort born in 1901 are assumed to start work.. The Congressional Budget Office (2004) estimates that the long run real interest rate in the United States is $r = 0.033$ and the long run rate of real wage growth is $\sigma = 1.27\%$ per annum. This means that real wages in 1971 (when the cohort born in 1951 start work) are 189 and rise to 334 in 2016 (when the cohort born in 1996 start work). We take the rate of time preference to be $\delta = 0.02$ so that the individual's optimal consumption grows over time. Table 2 summarizes the parameter values we use in the calibration.

The first order condition for retirement is $v(R) = u'(c(R))w(R)$.

Hence we require

$$d\left(\rho + ae^{bR}\right) = w(0)e^{\sigma R}\left[c(0)e^{\frac{(r-\partial)}{\beta}R}\right]^{-\beta} = w(0)e^{(\sigma+\delta-r)R}c(0)^{-\beta} \qquad (24)$$

In order to solve equation (24) for the optimal retirement age R we to know for the initial level of consumption $c(0)$. To find this we apply the budget constraint

$$\int_0^T e^{-rt}\mathrm{s}(t)c(t)dt = \int_0^R e^{-rt}\mathrm{s}(t)w(t)dt \qquad (25)$$

Which implies

$$c(0)\int_0^\infty e^{-\left(r+\rho-\frac{(r-\partial)}{\beta}\right)t-\frac{a}{b}\left(e^{bt}-1\right)}dt = w(0)\int_0^R e^{-(\rho+r-\sigma)t-\frac{a}{b}\left(e^{bt}-1\right)}dt \qquad (26)$$

We can use (26) to replace $c(0)$ in (24) with an expression that contains the retirement age R. We can the solve (24) for R and then using (26) solve for $c(0)$ which combined with the growth rate of consumption from (21) gives us the entire time path of consumption. The complexity of equations (24) and (26) makes it difficult to obtain a closed form solution for the retirement age and consumption path. We explain the details of our solution method in Web Appendix 6.

Table 3 gives the calibrated retirement age in our model using different mortality schedules (after age 20) and initial wage levels. Moving along the first row of Table 2 we see the effect of changing the initial wage level, holding the survival schedule steady at that experienced by the 1901 birth cohort. By 1951 initial wages were 1.89 times greater than in 1901 while in 1996 they were 3.34 times larger. We see in Table 3 that higher lifetime incomes and consumption will lead to earlier retirement. We estimate the effect of rising income levels, holding life expectancy constant, would have reduced the retirement age to 59.3 for the cohort born in 1951 and will reduce it further to 54.5 for the cohort born in 1996. These estimates imply that a doubling of lifetime income reduces the retirement age by just over 5 years.

If instead we start with the 1901 survival schedule and 1901 real wages of 100 and then move down along the first column of Table 3 we see the effect of longer life spans. At the 1901 survival schedule total life expectancy conditional on reaching age 20 was a further 41.9 years. At the 1951 survival schedule life expectancy conditional on survival to age 20 rises to 49.4 years, a gain of 7.5 years. We estimate that this gain in life expectancy would lead to a rise in the retirement age of 3 years to an age of 68.1. We further estimate that a man living with the 1996 cohort survival schedule but 1901 cohort income would have a life expectancy conditional on reaching age 20 of 74.3 and would increase his retirement age to 70.3. These results imply that each increase in a year of life expectancy at age 20 increases the retirement age by about 0.4 years. Moving along the rows of Table 3 gives the effect of increases in income while move down a column gives the effect of longer lifespan. The diagonal (in bold) gives the overall effect in different cohorts that have differences in both survival schedules and income levels.

Table 4 gives the calibrated savings rate at age 20 as a percentage of initial income for each calibration. As we can see an increase in income levels leads to higher savings rates while longer life spans lower the saving rate.

Table 5 gives the actual median retirement age over the period 1960-2010 based on data from Gendell and Siegel (1992), and Gendell (2008). This is a mixture of the retirement behavior of different cohorts in the year, but the 1960-65 figure corresponds roughly with the 1901 birth cohort while the 2005-2010 figure reflects the behavior of the 1951 birth cohort. The calibrated reduction in the retirement age for the 1951 birth cohort relative to the 1901 cohort matches closely the observed reduction in age of retirement.

6. Conclusion

Our model demonstrates two major, long-term influences on the optimal age at retirement and on savings behavior. First, at higher levels of lifetime wages, the desire for increased leisure leads to early retirement funded through a higher savings rate. Second, longer life spans and healthier lives lead to a less than proportional increase in working life and lower savings rates. Calibration results indicate that historically the effect of higher wages has dominated leading to a secular trend towards early retirement and higher cohort savings.

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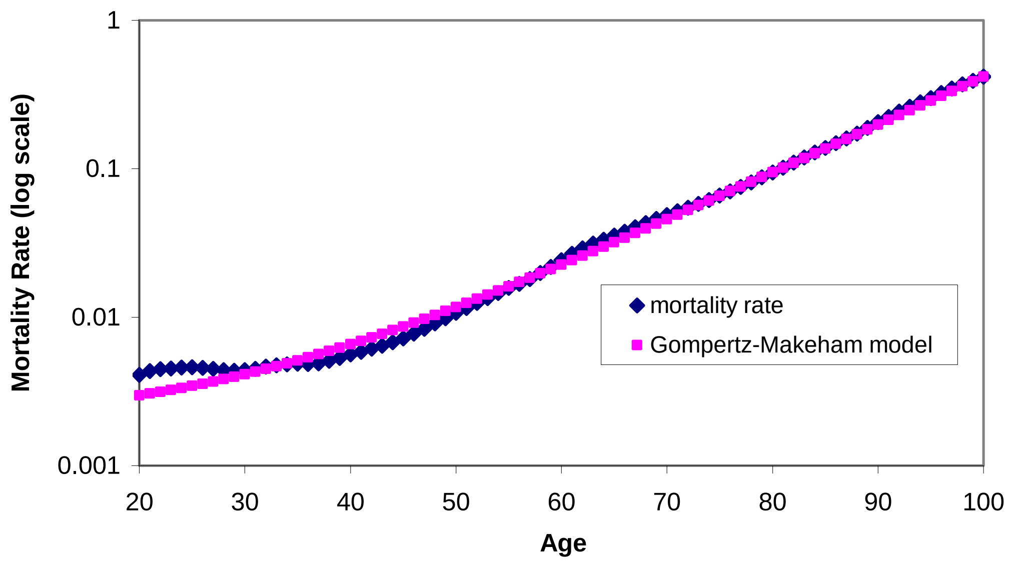
Figure 1
Adult Mortality Rates of the 1901 Birth Cohort



Source: Berkeley Mortality Database

Table 1
Estimated Gompertz-Makeham Mortality Parameters

	Birth cohort 1901	Birth cohort 1951	Birth cohort 1996
ρ x100	0.195* (0.045)	-0.134 (0.078)	-0.171* (0.065)
a x100	0.103* (0.004)	0.059* (0.005)	0.051* (0.004)
b	0.075* (0.0005)	0.080* (0.001)	0.079* (0.001)
N	80	80	80
R^2	0.999	0.997	0.997

Based on mortality rates at ages 20-100 from the Berkeley Mortality Database. Mortality at later ages for the 1951 cohort an at all ages for the 1996 cohort are projected.
* Statistically significant at 5% level.

Table 2
Parameters Used in Calibration

Interest Rate	r	0.03
Wage Growth	σ	0.0127
Rate of Time Preference	δ	0.03
Disutility of Work	d	0.87
Coefficient of Relative Risk Aversion	β	2

Table 3
Calibrated Retirement Ages

Cohort Birth Year	Life expectancy at age 20	Wage in 1921	Wage in 1971	Wage in 2016
		100	189	334
1901	41.9	**65.1**	59.3	54.5
1951	49.4	68.1	**62.3**	57.6
1996	54.3	70.3	64.4	**59.5**

Life expectancy at age 20 is based on data and projections from the Berkeley Mortality Database. We assume each cohort starts working at age 20. Wages in 1971 and projected wages in 2016 relative to 1921 are based on a long run rate of wage growth of 1.27% per annum.

Table 4
Calibrated Savings Rates at Age 20

Cohort Birth Year	Life expectancy at age 20	Wage in 1921	Wage in 1971	Wage in 2016
		100	189	334
1901	41.9	**18.5**	28.9	38.0
1951	49.4	13.0	**23.3**	32.2
1996	54.3	10.9	24.2	**30.3**

Life expectancy at age 20 is based on data and projections from the Berkeley Mortality Database. We assume each cohort starts working at age 20. Wages in 1971 and projected wages in 2016 relative to 1921 are based on a long run rate of wage growth of 1.27% per annum.

Table 5
Median Retirement Age

1960-1965	65.2
1965-1970	64.2
1970-1975	63.4
1975-1980	63.0
1980-1985	62.8
1985-1990	62.6
1990-1995	62.4
1995-2000	62.0
2000-2005	61.6
2005-2010	61.6

Sources: Gendell and Siegel (1992), Gendell (2008).