Noname manuscript No.
(will be inserted by the editor)

Voting rules and efficiency in one-dimensional bargaining games with endogenous protocol

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December 2010

Abstract We analyze the incentives of agents to invest in proposal rights in a one-dimensional bargaining game where agents preferences over social outcomes are single-peaked. We relate these incentives to the quota rules that are required to implement agreements. When the contest assigns persistent recognition probabilities, we find that extreme agents invest more and that simple majority reduces the total investments and, hence, inefficiency. In particular, we show for quadratic utility functions that the incentives to invest increase with the quota up to a threshold. In case that the contest recurs each period, multiple equilibria are obtained, with the particularity that only one agent controls the agenda of the bargaining process.

Keywords: One-dimensional bargaining, proposal rights, contests, quota rules, protocol.
JEL Classification: C78

1 Introduction

Proposal rights have been shown to be crucial in determining political power in collective negotiations over a divisible good (see e.g., Kalandrakis, 2006). However, Eraslan (2002) and Breitmoser (2010) show that their effects are limited in games with random proposers and majority bargaining. In these games, there is a wide range of recognition probabilities that yield an egalitarian expected outcome. Moreover, this range decreases with the majority requirement, implying that the profitability of being the proposer clearly depends on the consensus required to reach agreements.

In this paper, we study the relationship between proposal rights (or protocol) and voting rules in the context of a simple one-dimensional bargaining game. Specifically, we consider a contest game where agents invest in order to increase their probabilities of being the proposer in the ensuing bargaining game. Thus, the bargaining protocol emerges endogenously. It is known in this setup, that investments are never profitable when the core comprises a unique allocation and agents are

We acknowledge financial support from the Generalitat de Catalunya through grant SGR2009-1051 and Ministerio de Ciencia y Tecnologia through grant ECO2009-06953.

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sufficiently patient (see Cho and Duggan, 2009).[1] However, when the consensus requirement is higher than simple majority, the core is enlarged and investments become profitable. Interestingly, this profitability is positively related to the distance between the equilibrium outcome and the peak of the player. Hence, investments of extreme players are, in equilibrium, larger that those of central agents. Moreover, although the core increases with the quota, profitability does not necessarily increase accordingly. In particular, when preferences are quadratic, there may exist a threshold quota above which proposal rights become less valuable, as the decisive players become stronger when the majority requirement increases.

Our contest game is similar to that in Evans (1997), who analyzes a transferable utility coalitional bargaining game, in which a contest determines the recognition probabilities in each period. He finds that (under certain restrictions) the set of pure stationary subgame perfect equilibrium outcomes is equal to the core. In a pure bargaining setting, Yildirim (2007) shows that unanimity is optimal, in the sense that it minimizes investments in proposal rights. His results bear on the fact that probabilities are optimally (and indirectly through investments) selected in each period, so that actual probabilities do not affect the next period expected outcome. Hence, increasing the proposal power affects only the probability of being the proposer and to obtain the prize (i.e, the residual after compensating a majority of players), but not the probability of being included in any winning coalition (which would be the case when recognition probabilities were persistent). As the prize decreases with the quota, the optimality of unanimity follows.[2] The assumption of recurrent contest is not innocuous. In the pure sharing problem, when recognition probabilities are persistent, their effects are limited, as it has been put forward by Eraslan (2002).

In our (non-transferable utility) model, the fact that there is a pre-bargaining contest or that such a contest is recurrent also affects the results. We can address the case of persistent probabilities by using existing results on the existence, uniqueness and asymptotic convergence of the set of acceptable outcomes in one-dimensional bargaining. In order to focus exclusively on the effects of the quota rule on the resources lost due to the contest game, we impose a symmetric population, which will induce (in a unique equilibrium) a symmetric distribution of investments. This will render the bargaining outcome independent of the consensus requirement, and therefore, the quota rules will affect only the efficiency properties of the equilibrium. While in case of persistent probabilities, we find that a unique bargaining outcome is attained, in the case of a recurrent contest, we show that any bliss point in the underlying core can be supported as the equilibrium outcome when impatience vanishes. The salient property of any of these equilibria is that a single agent controls the agenda of negotiations.

In the next section, we present the model and results for the game with persistent recognition probabilities, i.e., when the protocol is determined at the beginning of the game and it persists over time. Section 3 considers the situation with recurrent contests, i.e., when recognition probabilities are determined via a contest in each period. Section 4 concludes.

2 The model with persistent recognition probabilities

Consider a finite set of n agents $I \subset [0,1]$ that are posed to select an alternative $x \in X = [0,1]$, where $\{0,1\} \in I$. The players in I negotiate over discrete time, $t = 0, 1, 2....$, according to a procedure that combines alternating proposals and voting. In each period $t \geq 0$, one player is randomly selected as the proposer according to a protocol $\mathbf{p}$. Then, she proposes an outcome $x \in X$, and all other players

[1] There are lots of papers that address the convergence of the stationary equilibrium outcomes to the core. See for instance Perry and Reny (1994), Lagunoff (1994), Moldovanu and Winter (1995), Evans (1997) or Banks and Duggan (2000).

[2] Yildirim (2010) also analyzes the case with persistent recognition probabilities and unanimity.

reply sequentially (in any fixed order) with acceptance or rejection. The proposal is approved if at least a proportion $q \in \{(\lfloor n/2 \rfloor + 1)/n, (\lfloor n/2 \rfloor + 2)/n, ..., 1\}$ of the players (including the proposer) accept it, where $\lfloor z \rfloor$ denotes the integer part of $z \in \mathbb{Q}$. In this case, the proposed alternative is implemented and the game ends; otherwise, the game moves to period $t+1$, where a new proposer is (randomly) selected, and so on.

Prior to the negotiation stage, a contest game determines the bargaining protocol $\mathbf{p}$. Agents simultaneously decide whether to enter the contest (by paying a small participation fee) and the amount/effort invested to increase the probability of being the proposer in the ensuing negotiation stages. Let $\widehat{I} \subset I$ denote the set of players that decided to enter the contest, and let $a_i \geq 0$ denote the investment of agent $i \in \widehat{I}$. We refer to the decision of a non-participating agent $i \in I \backslash \widehat{I}$ as $a_i = \varnothing$. Investment costs are given by $C(a_i) = \underline{a} + c(a_i)$ if $i \in \widehat{I}$ and zero otherwise, where $c : \mathbb{R}_+ \to \mathbb{R}_+$ is assumed differentiable, strictly convex and satisfying $c(0) = c'(0) = 0$, and $\underline{a}$ is a positive parameter, which is assumed sufficiently low.[3]

If $\widehat{I} = \varnothing$ then the game ends immediately, and all agents obtain zero payoffs. Otherwise, for any vector of investments $\mathbf{a} = (a_0, ..., a_1)$, a success function determines the protocol $\mathbf{p}(\mathbf{a}) = (p(0|\mathbf{a}), ..., p(1|\mathbf{a}))$ of the ensuing bargaining game. We assume that this function satisfies $p(i|\mathbf{a}) > 0$ for all $i \in \widehat{I}$, $p(i|\mathbf{a}) = 0$ for all $i \in I \backslash \widehat{I}$ and $\sum_{i \in I} p(i|\mathbf{a}) = 1$, where $p(i|\mathbf{a})$ is the probability that agent i is selected as the proposer in each period. We assume further that $p(i|\mathbf{a})$ is anonymous (with respect to i), continuously differentiable, strictly increasing and concave in $a_i > 0$. The vector of investments $\mathbf{a}$ induces a cumulative distribution of proposal rights, given by

$$G(z|\mathbf{a}) = \sum_{i \leq z} p(i|\mathbf{a}).$$

The instantaneous preferences of the agents over $x \in X$ are described by continuous and (twice) differentiable utility functions $u : [0,1] \times [0,1] \to \mathbb{R}_+$, with $u(x,i) = v(|x-i|)$ satisfying $v' < 0$ and $v'' \leq 0$ at any $x \neq i$, where $i \in I$ denotes both an agent and her peak.[4] Let $F : [0,1] \to [0,1]$ denote the cumulative distribution of peaks, which are assumed to be symmetrically distributed with respect to $1/2$. To simplify notation, we assume that all agents have different preferences.[5] Upon agreement on $x \in X$ in period t, player $i \in I$ obtains the payoff $\delta^t u(x,i) - C(a_i)$. Perpetual disagreement yields $-C(a_i)$ for all agents in $\widehat{I}$, and zero for others.

An individual *(pure) strategy* specifies the initial entry/investment decision in the contest stage, and a proposal and an acceptance/rejection rule for each subgame in the bargaining stage. According to a *stationary strategy*, each individual makes the same proposal whenever she is selected as the proposer, and uses the same acceptance/rejection rule in each subgame. A *stationary subgame perfect equilibrium (SSPE)* is a profile of stationary strategies that are mutually best responses in each subgame.

2.1 The bargaining game

Cardona and Ponsatí (2010) showed that, for any stationary protocol satisfying $p(i|\mathbf{a}) > 0$ for all agents, a unique (no-delay) subgame perfect equilibrium in the bargaining game is attained. In

[3] The threshold $\underline{a}$ is crucial in order to ensure existence of a stationary subgame perfect equilibrium in pure strategies in the simple majority bargaining game, as we remark later. Existence is the main problem when analyzing contests that yield perpetual proposal rights in a pure sharing bargaining game.

[4] We assume differentiability to simplify the exposition. In fact, continuity, which implies differentiability almost everywhere, would be sufficient.

[5] All results are maintained if we allow two or more players to have the same preferences, i.e., the same peak. However, the presentation would require a more complicated notation.

our setup, $p(i|\mathbf{a})$ is not necessarily strictly positive as the protocol is endogenously determined by the players. However, their results imply that for any quota rule, the SSPE is characterized by an approval set $[\underline{x}, \overline{x}] \subset [0,1]$, where $\underline{x}$ and $\overline{x}$ are determined by *two pivotal players*, l and r, satisfying $F(l) = 1-q$ and $F(r) = q$. Specifically, $\underline{x}$ and $\overline{x}$ solve the system of equations,

$$u(\underline{x}, r) = \delta\left[G(\underline{x}|\mathbf{a})u(\underline{x}, r) + \sum_{i\in(\underline{x},\overline{x}]} p(i|\mathbf{a})\, u(i,r) + (1-G(\overline{x}|\mathbf{a}))\, u(\overline{x}, r)\right],$$

$$u(\overline{x}, l) = \delta\left[G(\underline{x}|\mathbf{a})u(\underline{x}, l) + \sum_{i\in(\underline{x},\overline{x}]} p(i|\mathbf{a})\, u(i,l) + (1-G(\overline{x}|\mathbf{a}))\, u(\overline{x}, l)\right].$$

Moreover, as impatience vanishes, the approval set collapses to a unique point. We refer to this asymptotic outcome as the *equilibrium outcome* of the bargaining game, which we characterize in the next proposition. Although the results depend on the pivotal players l and r, in the remainder of this section, we use the symmetry assumption ($l = 1-r$) and present all results as a function of r only.

Proposition 1 *(Unique asymptotic outcome) As players become perfectly patient, the unique SSPE outcome of the bargaining game with protocol* $\mathbf{p}(\mathbf{a})$ *is given by* x^**, where*

1. *If* $p(i|\mathbf{a}) > 0$ *for some* $i \leq r$ *and* $p(j|\mathbf{a}) > 0$ *for some* $j \geq 1-r$*, then either*
   (a) $x^* \in [1-r, r]$ *solves*
   $$K(x,\mathbf{a},r) \equiv G(x)u_x(x,1-r)u(x,r) + [1-G(x)]u(x,1-r)u_x(x,r) = 0, \tag{1}$$
   or
   (b) *there exists a unique* $i^* \in I \cap [1-r, r]$ *such that* $K(x,\mathbf{a},r) > 0$ *for* $x \in [0, i^*)$ *and* $K(x,\mathbf{a},r) < 0$ *for* $x \in [i^*, 1]$*, and* $x^* = i^*$*.*
2. *If* $p(i|\mathbf{a}) = 0$ *for all* $i \leq r$ *then* $x^* = \min\{i \in I : p(i|\mathbf{a}) > 0\}$
3. *If* $p(i|\mathbf{a}) = 0$ *for all* $i \geq l$ *then* $x^* = \max\{i \in I : p(i|\mathbf{a}) > 0\}$*.*

Proof The proof of part (1) can be easily adapted from Predtetchinski (2007) or Cardona and Ponsatí (2008). While not assumed specifically in the latter, only $p(i|\mathbf{a}) > 0$ for some $i \leq r$ and $p(j|\mathbf{a}) > 0$ for some $j \geq 1-r$ is required. Parts (2) and (3) are proved in the Appendix. ■

The identity $K(x,\mathbf{a},r) = 0$ can be rewritten as follows:

$$G(x)\left(x - \frac{u(x,r)}{u_x(x,r)}\right) = [1-G(x)]\left(x - \frac{u(x,1-r)}{u_x(x,1-r)}\right),$$

where $x - \frac{u(x,r)}{u_x(x,r)}$ (resp. $x - \frac{u(x,1-r)}{u_x(x,1-r)}$) refers to the minimal (resp. maximal) proposal that agent r (resp. $1-r$) is willing to accept, when the acceptance set collapses to x in a limit SSPE. Thus, the above expression indicates that the equilibrium outcome x^* is such that all agents to the left of x^* make proposals that exactly compensate agent r, while agents to the right of x^* make proposals that exactly compensate agent $l = 1-r$.

Note that, if $0 < G(1/2|\mathbf{a}) < 1$ with n odd and a simple majority rule, the bargaining outcome is $x^* = 1/2$ independently of how proposal rights are distributed. Thus, the specific form of any non-degenerated (and possibly asymmetric) distribution of proposal rights does not affect the outcome under simple majority: the median voter's peak is obtained. However, by increasing the consensus requirements, the distribution of proposal rights has important effects on the resulting equilibrium outcome, as shown in the next simple example.

Example 1 Consider a three player game with $I = \{0, 1/2, 1\}$ and $u(x,i) = 1 - (x-i)^2$. When $p(i|\mathbf{a}) = 1/3$ for all i, the equilibrium outcome is 1/2 under any quota. If proposal rights are given by $p(0|\mathbf{a}) = 1/5, p(1/2|\mathbf{a}) = 1/5$ and $p(1|\mathbf{a}) = 3/5$, the outcome is $x^* = 1/2$ when $q = 2$, and $x^* \simeq 0.56$ when $q = 3$. Thus, the incentives to invest are clearly higher under unanimity.

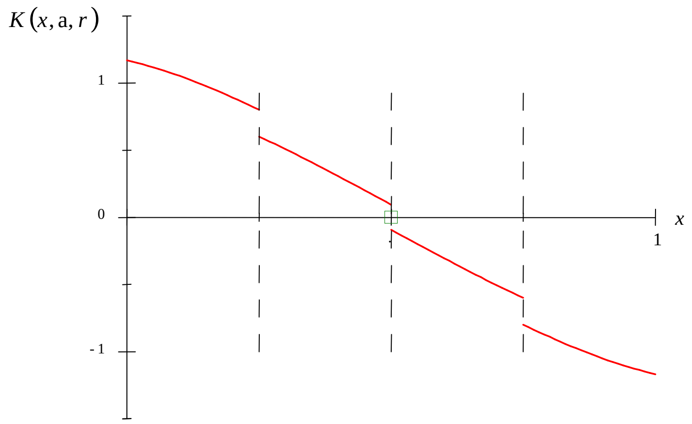
As we will see in the next subsection, this example is generic, showing that the incentives to invest are higher under strict super-majorities than under simple majority.

2.2 Investment decisions

As a consequence of Proposition 1.2 and 1.3, there is no SSPE with either $p\,(i|\mathbf{a}) = 0$ for all $i \leq r$ or $p\,(i|\mathbf{a}) = 0$ for all $i \geq 1 - r$. Otherwise, some agent (either $i \geq l$ in the first, or $j \leq r$ in the second case) could be better off by entering the contest and investing $a_i = 0$. Hence, we can restrict our attention to (limit) equilibria where $p\,(i|\mathbf{a}) > 0$ for some $i \leq r$ and $p\,(j|\mathbf{a}) > 0$ for some $j \geq 1 - r$, which are characterized by Proposition 1.1.[6] Obviously, if $p(i|\mathbf{a}) = 1$ for some $i \in I$, then $x^* = i$ is the unique outcome.

Investments affect the proposal rights continuously so that $K(x, \mathbf{a}, r)$ is continuous in $a_i \geq 0$, but not in x. In particular, K is discontinuous at x such that $p(x|\mathbf{a}) > 0$. This implies that marginal changes in the distribution of proposal rights do not necessarily alter the bargaining outcome at these points. For instance, when the outcome x^* is such that $\lim_{x \uparrow x^*} K(x^*, \mathbf{a}, r) > 0$ and $K(x^*, \mathbf{a}, r) < 0$, this outcome would remain after a marginal increase/decrease of any $a_i \geq 0$. As an example, the next picture reveals the discontinuity of the function $K(x, \mathbf{a}, r)$ at points x with $p(x|\mathbf{a}) > 0$.

Example 2 Consider $I = \{0, 1/4, 1/2, 3/4, 1\}$ with $u(x,i) = 1-(x-i)^2$, and assume $p(i|\mathbf{a}) = 1/5$ for all $i \in I$. The following picture depicts $K(x, \mathbf{a}, r)$ for $r = 3/4$ ($q = 4/5$)). The equilibrium outcome is given by $x^* = 1/2$.



Note the importance of the discontinuity of $K(x, \mathbf{a}, r)$ with respect to x. If we denote by $\Lambda_i\,(x, \mathbf{a}, r)$ the marginal effect of a_i on the bargaining outcome, it is immediate that $\Lambda_i\,(x, \mathbf{a}, r)$

[6] Note that, in these cases, the equilibrium outcome of the bargaining game satisfies $K(x^*, \mathbf{a}, r) \leq 0$ and $K(x^* - \varepsilon, \mathbf{a}, r) > 0$ for any $\varepsilon > 0$. Moreover, $K(x, \mathbf{a}, r)$ is continuous in x from the right.

may be equal zero for some agents when $K(x,\mathbf{a},r)$ is discontinuous at x; i.e., when $p(x|\mathbf{a}) > 0$. Hence, there are no (marginal) incentives for these agents to invest. On the other hand, when $p(x|\mathbf{a}) = 0$, the equilibrium must solve equation (1), and $K(x,\mathbf{a},r)$ is continuous and differentiable at this point. Implicit differentiation of equation (1) yields[7]

$$\Lambda_i\left(x,\mathbf{a},r\right) = \left.\frac{\partial x}{\partial a_i}\right|_{K(x,\mathbf{a},r)=0} = -G_{a_i}\left(x|\mathbf{a}\right)\frac{K_1\left(x,\mathbf{a},r\right)}{K_2\left(x,\mathbf{a},r\right)},$$

where

$$\begin{aligned} K_1\left(x,\mathbf{a},r\right) &= u_x\left(x,1-r\right)u\left(x,r\right) - u\left(x,1-r\right)u_x\left(x,r\right), \textit{and} \\ K_2\left(x,\mathbf{a},r\right) &= G\left(x|\mathbf{a}\right)u_{xx}\left(x,1-r\right)u\left(x,r\right) + \\ &\quad + \left[1-G\left(x|\mathbf{a}\right)\right]u\left(x,1-r\right)u_{xx}\left(x,r\right) + u_x\left(x,1-r\right)u_x\left(x,r\right). \end{aligned}$$

Using the concavity assumption of u and the fact that the equilibrium outcome satisfies $x \in [1-r, r]$, we obtain that $K_1\left(x,\mathbf{a},r\right) \leq 0$ and $K_2\left(x,\mathbf{a},r\right) < 0$. In particular, when $r > 1/2$, $\Lambda_i\left(x,\mathbf{a},r\right)$ has a positive (negative) sign if $G_{a_i}\left(x|\mathbf{a}\right) < (>)\, 0$ for $x \in [1-r, r]$. Moreover, $\Lambda_i\left(x,\mathbf{a},r\right)$ is zero when $r = 1/2$. Thus, independently of the fact that $K(x,\mathbf{a},r)$ is continuous or not at x, investment decisions will affect the outcome only when $r > 1/2$. The next proposition describes the equilibrium outcome for simple majority and an odd number of players.

Proposition 2 *Under simple majority and n odd (i.e., $r = 1/2$), the equilibrium outcome is given by $x^* = 1/2$ and either (a) $p(1/2|\mathbf{a}) = 1$ or (b) $p(i|\mathbf{a}) = p(j|\mathbf{a}) = 1/2$ for some i, j with $i < 1/2 < j$. In any case, $a_z = \varnothing$ for all other agents.*

Proof Note first that x^* must be equal to $1/2$, since otherwise agent $i = 1/2$ can induce it by choosing $a^*_{1/2} = 0$, which would be profitable when $\underline{a}$ is sufficiently small. Assume $a_{1/2} \geq 0$, in which case the equilibrium outcome is given by $x^* = 1/2$, independently of the distribution of proposal rights. Thus, $a_j = \varnothing$ is optimal for all other agents $j \neq i$. In this case, $a^*_{1/2} = 0$ is also optimal. In case that $a_{1/2} = \varnothing$ and $p\left(i|\mathbf{a}\right) > 0$, $p(j|\mathbf{a}) > 0$ for i, j satisfying $i < 1/2 < j$, it must be that $a_i = a_j = 0$ and that (optimally) no other agent enters the contest, since no change in the proposal rights will change the outcome because $0 < G\left(1/2|\mathbf{a}\right) < 1$. Moreover, $a_i = a_j = 0$ are also optimal, since (1) any increase on investments does not alter the outcome and, (2) by selecting $a_i = \varnothing$ (resp. $a_j = \varnothing$) the bargaining outcome would be $x^* = j$ (resp. $x^* = i$), in which case (for $\underline{a}$ is sufficiently small) the payoff of agent i (resp. j) decreases. ■

The last proposition implies that, under simple majority the inefficiency generated by the contest game is minimized, in the sense that at most $2\underline{a}$ resources are lost due to the contest competition.[8] This first-best outcome does not hold when the core of the underlying cooperative game is not a singleton, i.e., when strict super-majorities are required to implement policies. In these cases, the incentives to invest are positive as far as they are effective, i.e., when $\Lambda_i\left(x^*,\mathbf{a},r\right) > 0$. Although investments are endogenous and we might have $p(x^*|\mathbf{a}) > 0$, implying $\Lambda_i\left(x^*,\mathbf{a},r\right) = 0$, this can never happen in equilibrium if $r > 1/2$. In other words, any increase in a_i affects the bargaining outcome when $i \neq x^*$. Moreover, although the marginal effect of a_i on x^* is the same for all $i < x^*$ (resp. $i > x^*$), the utility gains from a change in x^* differs across these agents due to the concavity of their utilities. In particular, under strict concavity, extreme agents have more to gain (resp. lose). This finding is formalized in the next proposition.

[7] As $p\left(i|\mathbf{a}\right) = 0$ for $i = x$, we have that $G_x\left(x|\mathbf{a}\right) = 0$.

[8] Notice the importance of the threshold $\underline{a}$. When simple majority is required, then $x^* = 1/2$ as far as there are two players i, j, with $i < 1/2 < j$, such that $p(i|\mathbf{a}), p(j|\mathbf{a}) > 0$. If such a threshold was not imposed, both agents would select the smallest $a_i > 0$. Hence, by continuity, there would be no optimal strategy.

Proposition 3 *If $r > 1/2$ there is a unique SSPE, which yields $x^* = 1/2$. Moreover, if u is strictly concave then the SSPE investments are strictly increasing with the distance of the agents to $1/2$, with $a_{1/2} = \varnothing$ if the number of agents is odd.*

Proof The proof is based on Lemmata 1 to 4 proved in the Appendix. Using Lemmata 1 and 2 it is shown that the equilibrium satisfies $p\left(x^*|\mathbf{a}\right) = 0$. Lemmata 3 and 4 show that $a_i^* = a_{1-i}^*$ for all $i \in I$ so that the distribution $G\left(x|\mathbf{a}\right)$ is symmetric (as the success function is assumed anonymous), which implies, using (1), that $x^* = 1/2$. Moreover, K is continuous at x^*. Assume $a_i^* \leq a_j^*$ for some $i > j > x^* = 1/2$, which implies (due to the concavity of the success function) that $\Lambda_i\left(x^*, \mathbf{a}, r\right) \geq \Lambda_j\left(x^*, \mathbf{a}, r\right)$. Moreover, under strict concavity of utilities, we have that $u_x\left(x^*, i\right) > u_x\left(x^*, j\right)$. The first order conditions in the investment stage are given by

$$u_x\left(x^*, i\right) \Lambda_i\left(x^*, \mathbf{a}, r\right) = c'\left(a_i^*\right) \text{ for any } i \in \widehat{I}.$$

Therefore, we must have $c'\left(a_i^*\right) > c'\left(a_j^*\right)$, contradicting $a_i^* \leq a_j^*$. Lemma 6 in the Appendix proves existence. ■

Besides the fact that in any equilibrium the median voter (when exists) does not invest, and that under strict concavity of preferences extreme agents invest more than more central ones, some remarks are in place:

1. When $\Lambda_i\left(x^*, \mathbf{a}, r\right) > 0$ then clearly agent $j > i > x^*$ benefits from a_i, i.e., investments have a public good component so that free riding behavior appears.
2. Under tent-shaped utilities, i.e., $v'' = 0$, the gains from investments are the same for any $j, i > x^*$. Thus, all agents invest the same amount.
3. Although extreme agents' acceptance is not required when $q < 1$, they affect the outcome in their favor by increasing their proposal rights.

As the core of the underlying cooperative game is enlarged with higher quotas and $G_{a_i}\left(x^*|\mathbf{a}\right)$ does not depend on the latter, the intuition suggests that investments should increase with the quota. However, this is not necessarily the case. In particular, when agents have quadratic utilities, there is a threshold $\overline{r}$ such that incentives to invest decrease in r for $r > \overline{r}$.

Proposition 4 *For the quadratic utility function $u(x, i) = 1 - (x - i)^2$, there is a threshold $\overline{r} < 1$ such that, in equilibrium, the profitability of investments increases in r up to $\overline{r}$ and decreases for $r > \overline{r}$.*

Proof By differentiating $\Lambda_i\left(x^*, \mathbf{a}, r\right)$ at $x^* = 1/2$, we obtain

$$\frac{\partial \Lambda_i\left(1/2, \mathbf{a}, r\right)}{\partial r} = 2G_{a_i}\left(1/2|\mathbf{a}\right) \frac{1 - 8r\left(1 - r\right)\left(9 - 2r + 2r^2\right)}{\left(-4r^2 + 4r - 5\right)^2},$$

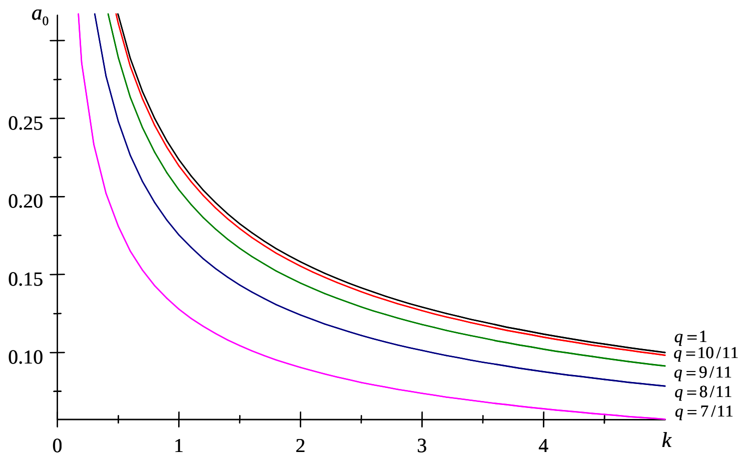
i.e., there exists a unique $\overline{r} \in [1/2, 1]$ such that $\dfrac{\partial \Lambda_i}{\partial r} = 0$. Thus, the incentives to invest increase in q until $\overline{q}$ satisfying $F(\overline{q}) = \overline{r}$ and decrease thereafter. ■

Any increase in r (or q) has two effects. First, the underlying core is enlarged, which increases the profitability of investments. Secondly, the decisive players, r and l, become more extreme so that, for strictly concave utilities, their requirements to accept a proposal are higher. Thus, the profitability of investments is reduced. As a consequence, efficiency loses are not necessarily monotonic with the quota.

Remark 1 The previous Proposition should be carefully interpreted. It says that the profitability of investments increases up to a threshold, which is smaller than 1. Note, however, that for some populations there is no q where $\overline{r}$ is the pivotal agent. In these cases, it is possible that the profitability of investments increases monotonically with the quota.

As an example, a numerical simulation is provided.

Example 3 Let $I = \{0, 0.1, .., 0.9, 1\}$, i.e., $n = 11$, $C(a) = 10^{-3} + (k/2)\, a^2$ and $p(i|\mathbf{a}) = \frac{a_i + \underline{a}}{\sum_{j \in \hat{I}} (a_j + \underline{a})}$. The parameter k indicates how costly investments are. We depict the equilibrium investments of agent $i = 0$. Other agents behave similarly. For any k, investments increase with the quota, that ranges from $q = 7/11$ to $q = 1$.While it is easily checked that $\Sigma_{i \in I} u(x^*, i) = 9.9$, in the table we show the efficiency loses as a function of q, which, in this particular example, turn out to be independent of k.



Investment of the agent $i = 0$ under alternative quota rules.

q	6/11	7/11	8/11	9/11	10/11	1
Total costs $(\cdot 10^{-3})$		35.9	67.7	91.8	106.2	110

3 The model with recurrent contests

As we noticed in the introduction, it is crucial for our results in the previous section that the recognition probabilities persist in all bargaining subgames. Equilibria may change dramatically when recognition rights vanish after a rejection and contests recur each period. Formally, the game differs from that of Section 2 only by the fact that agents take investment decisions in each period. Accordingly, a *stationary strategy* prescribes the same investment decision in each contest.

In this setting, the expected continuation payoffs are not affected by changing actual investments. Thus, the incentives to invest are smaller than under perpetual recognition probabilities. On the other hand, the fact that an agent makes positive (stationary) investments implies she has to support the same costs in the next period. Hence, her expected continuation payoff is reduced, implying that her individual acceptance set is enlarged. If this was reflected in a larger (collective) acceptance set, proposal rights would be more valuable. Although any of these two effects may dominate and, therefore, either (a) the incentives to invest vanish or (b) future (stationary) investments reinforce the profitability of actual investments, we show that, for sufficiently patient players, there exist multiple SSPE where only the first effect appears. The property of these equilibria is that only one agent $j \in [l, r]$ enters the contest and invests $a_j = 0$ in each period. Hence, the acceptance set collapses to j when agents are patient enough. Moreover, as actual investments do not affect the latter acceptance set, no other agent will have incentives to invest. Consequently, agent j will choose $a_j = 0$.

Proposition 5 *In a game with recurrent contests, any $j \in I$ with $j \in [l, r]$ can be supported as an SSPE outcome for sufficiently patient agents.*

Proof Consider $a_j = 0$ for some $j \in [l, r]$ and $a_i = \varnothing$ for all $i \neq j$. Expected utilities are given by $U_i = u(i, j)$ for all $i \in I$. It is clear that $a_j = 0$ is optimal, as

$$1 - \underline{a} \geq \delta (1 - \underline{a}).$$

Consider now the strategies of any other agent i, which we assume w.l.o.g. $i < j$. By entering the contest, we have that $p\left(i|\mathbf{a}'\right) \geq 1/2$ and $p\left(j|\mathbf{a}'\right) > 0$. In case that i is recognized as the proposer, she can get $x_i \geq i$ satisfying

$$u(x_i, r) = v(|x_i - r|) = \delta v(|j - r|),$$

as $p(j|\mathbf{a}) = 1$ in the next period. Moreover, for any $\varepsilon_i > 0$ there is $\delta(\varepsilon_i) \in (0, 1)$ such that $j - x_i < \varepsilon$ for all $\delta > \delta(\varepsilon_i)$. In these cases, the expected payoff of agent i is given by

$$p\left(i|\mathbf{a}'\right) v(|x_i - i|) + \left(1 - p\left(i|\mathbf{a}'\right)\right) v(|j - i|) - C(a_i).$$

Choose ε_i such $v(|x_i - i|) - v(|j - i|) < \underline{a}$. Hence,

$$\begin{aligned} & p\left(i|\mathbf{a}'\right) v(|x_i - i|) + \left(1 - p\left(i|\mathbf{a}'\right)\right) v(|j - i|) - C(a_i) \\ & < p\left(i|\mathbf{a}'\right) [\underline{a} + v(|j - i|)] + \left(1 - p\left(i|\mathbf{a}'\right)\right) v(|j - i|) - \underline{a} \\ & < \left(p\left(i|\mathbf{a}'\right) - 1\right) \underline{a} + v(|j - i|) < v(|j - i|), \end{aligned}$$

i.e., no agent $i \in I - \{j\}$ benefits from entering the contest when $\delta > \delta(\varepsilon_i)$. Hence, by selecting $\varepsilon = \min\left\{\varepsilon_j : j \in I\right\}$ the result follows. ■

Remark 2 It appears that the forces driving the results in the recurrent contests game are similar to those in Che and Sákovics (2004). The stationarity restriction on the strategies is so strong that it determines the expected continuation outcome of the game. In particular, when only one agent enters the contest then, for sufficiently patient players, the expected continuation outcome is a singleton. Thus, no other agent has incentives to invest, as she cannot affect the acceptable set (because of stationarity). This, in turn, is consistent with the continuation acceptance set being singleton, and the fact that only one agent makes proposals.

Although we analyze a symmetric environment with contests recurring each period, the equilibria are not necessarily symmetric. Interestingly, any peak in the underlying core of the game can be supported as an SSPE outcome, with the particularity that only one agent makes proposals (thus, minimizing inefficiencies) in the bargaining game. This contrast starkly with the case where recognition probabilities are persistent, where a unique outcome arises at the cost of inefficient investment decisions.

The multiplicity result for recurrent contests complements Evans (1997), who shows that in a transferable utility coalitional bargaining game with no discount (or little discount and discrete offer space), the core contains all its pure SSPE outcomes.[9]

4 Final remarks

This work proves the (asymptotic) existence and uniqueness of the SSPE outcome in a simple symmetric one-dimensional bargaining game, where previous to negotiations agents compete to become the proposers and recognition probabilities are persistent over time. We have shown that in the unique SSPE extreme agents invest more than the more central ones. Moreover, although the consensus requirement governing decisions does not affect the bargaining outcome (in our symmetric setup), it does affect investments and therefore the efficiency of the decision process. We have shown that simple majority reduces the inefficiency that the contest creates. Moreover, inefficiency loses are not necessarily monotonic in the majority requirement. We give an example, where the inefficiency increases up to a threshold and declines thereafter.

In situations where the contest is recurrent, we showed that multiple equilibria exist. In this case, the stationarity assumption is so strong that the acceptance set may collapse to a unique outcome when *pivotal players* do not enter the contest(s). Moreover, as this acceptance set does not depend on the actual investment decisions (because of the stationary assumption), agents can only marginally change this outcome when proposing. Therefore, the incentives to become the proposer vanish. In fact, all peaks in the interior of the underlying core constitute an SSPE outcome when agents are patient enough, with the particularity that only one agents enters the contest and becomes the dictator in the ensuing bargaining game.

It is worth to stress one aspect of our model of persistent recognition probabilities that is in contrast with the pure bargaining setting. In our public good setting, agents may benefit from investments made by other agents with similar preferences. However, when evaluating the gains from their investments, they do not consider the positive externalities that they generate. Allowing agents on the same side of $1/2$ (or agents whose peaks are close) to coordinate their investment decisions would increase the inefficiency generated by the contest.

The success function considered in this paper has some properties that deserve separate comments. In particular, it captures the idea that agents not participating in the contest have zero probability of being the proposer. Moreover, a minimal threshold has to be imposed to avoid non-existence problems in the simple majority case. Other success functions that assign a positive probability to all players, independently of their investment decisions, would avoid such an existence problem in the simple majority case. However, existence problems would appear with strong consensus requirements: A positive probability for all players would imply that in any symmetric equilibrium x^* the function K is not continuous at this point when the number of players is odd. Consequently, all agents would have to invest zero, which in turn would be inconsistent with an optimal investment decision (for $\underline{a}$ small enough). In these circumstances, the analysis would be significantly more complicated, as we had to consider mixed strategy equilibria.

Acknowledgments: We thank Lluis Bru for a helpful discussion.

[9] In fact, when there is positive discounting of payoffs, Evans (1997) has to introduce that the set of feasible investments, as well of the set of feasible agreements, is discrete in order to avoid non existence of pure strategies SSPE. In our model, we avoid such an existence problem by introducing a small entry cost, which has the same effects than imposing a discrete set of feasible investments.

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Appendix

Proof (Proof of Proposition 1) From Cardona and Ponsatí (2010), when $\delta < 1$ the acceptance set is given by a compact interval $[\underline{x}, \overline{x}]$ satisfying

$$u(\underline{x}, r) = \delta U_r \text{ and } u(\overline{x}, l) = \delta U_l,$$

where

$$U_i = G(\underline{x}|\mathbf{a})u(\underline{x}, i) + \sum_{j \in (\underline{x}, \overline{x}]} p_j u(j, i) + (1 - G(\overline{x}|\mathbf{a}))\, u(\overline{x}, i)\,.$$

Assume case (2). I.e., $G(j|\mathbf{a}) = 1$ for some $j \leq l$ and $p(j|\mathbf{a}) > 0$. In this case, $\underline{x}$ must be smaller than j since otherwise $u(\underline{x}, r) = \delta u(\underline{x}, r)$, which is a contradiction. Moreover, $\overline{x} \geq j$ since otherwise all agents $i \geq j$ would accept $\overline{x} = j$, contradicting that $\overline{x}$ is the largest acceptable proposal. Thus, $j = x^* = \max\{i \in I : p(i|\mathbf{a}) > 0\} \in [\underline{x}, \overline{x}]$ for all $\delta < 1$. Note also that $(1 - G(\overline{x}|\mathbf{a})) = 0$. Moreover, for any $\varepsilon > 0$ there exists $\overline{\delta} \in (0, 1)$ such that $x^* - \underline{x} < \varepsilon$ for all $\delta > \overline{\delta}$: Since $u(x^*, r) > u(i, r) > u(\underline{x}, r)$ for any $i \in (\underline{x}, x^*)$ and

$$u(\underline{x}, r) = \delta \left[G(\underline{x}|\mathbf{a})u(\underline{x}, r) + \sum_{j \in (\underline{x}, x^*]} p(j|\mathbf{a})u(j, r) \right],$$

we must have that $x^* - \underline{x}$ approaches to zero as δ approaches to 1.

Case (3) can be proved similarly. ■

Lemma 1 *In any equilibrium outcome x^* we must have that either $p(i|\mathbf{a}) = 0$ or $p(i|\mathbf{a}) = 1$ when $i = x^*$.*

Proof Assume not. I.e., $0 < p(i|\mathbf{a}) < 1$. Assume w.l.o.g. $x^* \leq 1/2$. Since $p(i|\mathbf{a}) \neq 0$ the function $K(x, \mathbf{a}, r)$ is discontinuous at x^*. We distinguish three cases: (1) $\lim_{x \uparrow x^*} K(x, \mathbf{a}, r) > 0$ and $K(x^*, \mathbf{a}, r) = 0$, (2) $\lim_{x \uparrow x^*} K(x, \mathbf{a}, r) = 0$ and $K(x^*, \mathbf{a}, r) < 0$, and (3) $\lim_{x \uparrow x^*} K(x, \mathbf{a}, r) > 0$ and $K(x^*, \mathbf{a}, r) < 0$. In the last case, we must have that $a_i = 0$ for all $i \in \hat{I}$, since any marginal change in a_i will not affect the equilibrium outcome. Thus, by the anonymity of the success function,

$$K(x^*, \mathbf{a}, r) \equiv \frac{n_L + 1}{n_L + n_R + 1} u_x(x^*, 1 - r) u(x^*, r) + \frac{n_R}{n_L + n_R + 1} u(x^*, 1 - r) u_x(x^*, r) < 0,$$

where n_L and n_R denote the agents in $\widehat{I}$ to the left and to the right of i, respectively. Note that, as $x^* \leq 1/2$, we must have $n_L + 1 \geq n_R$. Moreover, as the optimal strategy of agent $i = x^*$ is $a_i = 0$, it has to be that when she chooses $a'_i = \varnothing$ then the bargaining outcome also changes. I.e.,

$$K(x, \mathbf{a}', r) \equiv \frac{n_L}{n_L + n_R} u_x(x, 1 - r) u(x, r) + \frac{n_R}{n_R + n_L} u(x, 1 - r) u_x(x, r) > 0.$$

In case that $n_L - n_R \geq 0$, which implies

$$\frac{n_L + 1}{n_L + n_R + 2} \leq \frac{n_L}{n_L + n_R} \text{ and } \frac{n_R + 1}{n_R + n_L + 2} \geq \frac{n_R}{n_R + n_L},$$

we have that if some agent $j > i = x^*$ with $a_j = \varnothing$ changes to $a'_j = 0$, then we obtain

$$\frac{n_L + 1}{n_L + n_R + 1} u_x(x, 1 - r) u(x, r) + \frac{n_R + 1}{n_R + n_L + 1} u(x, 1 - r) u_x(x, r) \geq K(x, \mathbf{a}', r) > 0.$$

Thus, it must be that all agents $j > i$ enter the contest. This implies (as $x^* \leq 1/2$) $n_R \geq n_L$, and therefore (as $n_L + 1 \geq n_R$) $n_L = n_R$ and $i = x^* = 1/2$, contradicting $K(x, \mathbf{a}', r) > 0$. If $n_L - n_R < 0$, then we must have $n_L + 1 = n_R$, which implies $x^* = 1/2$, and therefore $K(x^*, \mathbf{a}, r) = 0$, which is a contradiction.

In case (1), we have that all agents $j > x^*$ must choose $a^*_j > 0$ (as K is continuous from the right, $c'(0) = 0$ and $\underline{a}$ is sufficiently small) and, therefore, a marginal increase in a_j increases the bargaining outcome. However, at the same time $a^*_j = 0$, since a marginal decrease in a_j does not alter the bargaining outcome. Thus, we get a contradiction. A similar argument excludes case (2). ■

Lemma 2 *There is an equilibrium outcome x^* with $p(x^*|\mathbf{a}) = 1$ only if $r = 1/2$ and $x^* = 1/2$.*

Proof Assume w.l.o.g. $i = x^* \leq 1/2$ with $p(x^*|\mathbf{a}) = 1$, and $r > 1/2$. This implies that, optimally, $a_i = 0$. Then, by entering the contest, any agent $j \neq i$ can propose with probability of at least $1/2$. Consider that agent $r > 1/2$ enters the contest. Then, the function $K(x, \mathbf{a}, r)$ is continuous in (i, r) so that there is a solution $x > x^*$, which benefits agent r. Thus, $r = 1/2$ is necessary. Note also that $r = 1/2$ implies that $x^* = 1/2$, since otherwise (using Proposition 1.2 or 1.3), agent r can obtain this outcome by entering the contest. ■

Lemma 3 *If $r > 1/2$, then in any SSPE, $u_x(x^*, i) \Lambda_i(x^*, \mathbf{a}, r) = c'(a^*_i)$ for all $i \in \widehat{I}$, implying $a^*_i > 0$.*

Proof Immediate by noting that $r > 1/2$ implies $p(x^*|\mathbf{a}) = 0$. ■

Lemma 4 *If $r > 1/2$ then $a^*_i = a^*_{1-i}$ for all $i \in I$.*

Proof Fix x^*, and assume w.l.o.g. $x^* \geq 1/2$. Thus,

$$\left| \frac{u_x(x^*, 1 - r)}{u(x^*, 1 - r)} \right| \geq \left| \frac{u_x(x^*, r)}{u(x^*, r)} \right|,$$

which is consistent with equation (1) only if $G(x^*|\mathbf{a}) \leq 1/2$. Let assume $a^*_i < a^*_{1-i}$ for some $i \in I$, with $i < 1/2$. This implies that $\Lambda_i(x^*, \mathbf{a}, r) u_x(x^*, i) = c'(a^*_i) < c'\left(a^*_{1-i}\right) = \Lambda_{1-i}(x^*, \mathbf{a}, r) u_x(x^*, 1 - i)$. This can be the case either if (a) $|u_x(x^*, i)| < |u_x(x^*, 1 - i)| \iff |x^* - i| < |x^* - (1 - i)|$, which is inconsistent with $i < 1/2$ and $x^* \geq 1/2$, or (b) $|\Lambda_i(x^*, \mathbf{a}, r)| < |\Lambda_{1-i}(x^*, \mathbf{a}, r)|$. However, as $G(x^*|\mathbf{a}) \leq 1/2$, we have that $G_{a_i}(x^*|\mathbf{a}) \geq -G_{a_{1-i}}(x^*|\mathbf{a}) > 0$, which is also a contradiction. ■

Proposition 6 (Existence) *The game with persistent recognition probabilities has an equilibrium.*

Proof For $r > 1/2$ we know that in any equilibrium $a_{1/2} = \varnothing$ if n is odd, since $a_i^* = a_{1-i}^*$ for all $i \in I$. If agents $i \in \ I - \{1/2\}$ choose their (strictly positive) investments according to their FOC condition[10]

$$u_x\left(x^*, i\right) \Lambda_i\left(x^*, \mathbf{a}, r\right) = c'\left(a_i^*\right),$$

the outcome is given by $x^* = 1/2$; and the SOC yields

$$u_x(x^*, i)\Lambda_{ii}(x^*, \mathbf{a}, r) - c''(a_i^*) = ((G_{a_i a_i}(x|a))/(G_{a_i}(x|a))) - c''(a_i^*) < 0.$$

Moreover, by strict convexity of c, the benefits from any (optimal) $a_i > 0$ are always greater than the mean costs when $\underline{a}$ is sufficiently small. ■

[10] Note that if n is even, then all agents are included.